MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Edward George Bloomfield Barlow, Ditton Lodge, Stourwood Avenue, Bournemouth;
Lieut. F. G. Cooper, R.N.R., H.M.S. "Ocean," 131 Sutton Court, Chiswick, W.; and
Edward Power, F.S.A., F.G.S., 16 Southwell Gardens, S.W., and Watership, Newbury, Berks,

were balloted for and duly elected Fellows of the Society.

One hundred and eight presents were announced as having been received since the last meeting, including, amongst others:—

Carl Friedrich Gauss Werke, Band vii. (presented by Prof. M. Brendel); Report of the Italian Commission on Observations of the total solar Eclipse of 1905 at Alcala Chivert, Spain (presented by the Catania Observatory); J. A. C. Oudemans, Mutual Occultations and Eclipses of the Satellites of Jupiter (presented by Prof. Nijland); Prague Observatory, Astronomische Beobachtungen, 1892–99, nebst Zeichnungen und Studien der Mondoberfläche, von L. Weinek (presented by the Observatory); Müller und Kempf, Photometrische Durchmusterung, Generalkatalog (presented by the Potsdam Observatory).

Astrographic Chart of the heavens: 40 charts presented by the Royal Observatory, Greenwich, and 20 charts by the San Fernando Observatory.

Baxendell's Observations of U Geminorum. Edited by H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. These observations are published at once for the reason mentioned on p. 119 of this volume, viz.:—the University of Utrecht has offered a prize (open to Dutch astronomers) for a dissertation on this Variable, and there have been requests made for original observations.

Mr Joseph Baxendell has now put into my hands the MS. observations made by his father, which go back to 1836. Inquiry having been made as to this particular star, it was elicited that these valuable observations of variables were for the most part not ledgered, but remained in the original observation books. Of these there are—

(*a*) Three foolscap volumes, 1836–48, 1848–56, and 1856–60,
(*b*) Seven small notebooks, 1861–77,
(*c*) A quarto notebook, 1877–88,

of which (*a*) and (*b*) represent work at Manchester, and (*c*) work at Southport.

2. The material (*b*) and (*c*) has all been now copied out in ledger form (under each star), and the ledgers are deposited in a building different from that containing the original books. The material (*a*) is more difficult to transcribe in ledger form. It is scarcely possible to hand it to anyone who is not familiar with variable star records, and procedure is under consideration. I take this opportunity of saying that I should be very glad of skilled volunteer assistance, at any rate in dealing with the copied ledgers for different stars, and perhaps with these early records also. If any variable star observer has leisure for work of the kind and would communicate with me, I should gratefully accept assistance in making this mass of valuable material ready for publication as soon as possible. Unaided, my work at it must necessarily be slow.

3. The observations of the elder Baxendell (*b* 1815–*d* 1887) divide themselves into two periods:—

1836–1877 at Manchester. "With his friend Mr Robert Worthington, of Crumpsall Old Hall, he erected the Crumpsall Observatory, where the large 13-inch reflector (the speculum of which he had himself cast, ground, and polished) was mounted, beside a small 5-inch equatorial refractor" (*Monthly Notices*, xlviii. p. 157). The former instrument is designated R(13) below, and the latter A. In *M. N.*, xviii. p. 11, the focal length of A is given as 70 inches. Besides these a 12-inch reflector belonging to Mr Williamson was used, and is designated R(12); and other small instruments—a 30-inch achromatic, a 22-inch achromatic, and a Tully telescope. These are called 30(*a*), 22(*a*), and T respectively. There are also B ("Mr Bowman's $7\frac{1}{4}$-inch") and C, a comet seeker of about $2\frac{1}{4}$ inches aperture. Mr Baxendell tells me that 22(*a*) was an excellent $2\frac{3}{4}$-inch by Dancer, who also made A.

1877–1888 at Southport. "In 1871 he was appointed superintendent of a meteorological observatory in Hesketh Park, fitted up and presented by John Fernley, Esq., formerly of Manchester; and in 1877 he erected his own private astronomical observatory in Birkdale, Southport, and resumed his observations of variable stars, etc. with a 6-inch equatorial refractor by Cooke & Sons, assisted for some years (previous to 1888) by his son" (*loc. cit.*, p. 158).

4. A careful note in the quarto MS. book (*c*) above gives a more detailed description of the Southport Observatory, and the following paragraphs concern us:—

"The observatory, the equatorial and micrometer, the portable transit instrument, the sidereal chronometer, and sidereal watch belong to Thomas S. Bazley, Esq., of Hatherop Castle, Fairford, Gloucestershire, who has kindly granted me the use of them so long as I may be able or inclined to make astronomical observations, and has also borne the expense of their removal from Hatherop Castle, and of the re-erection of the observatory, and re-mounting of the equatorial and transit instrument; and I think it a duty to make this record of his unlooked-for kindness and liberality, for which I am most sincerely and deeply grateful.

"The object-glass of the equatorial is 6 inches in diameter, and has a focal length of $87\frac{1}{2}$ inches. There are six negative eyepieces, a comet eyepiece, two reflection sun eyepieces, and three sunshades. The powers of the six negative eyepieces, as determined by Mr Bazley, are 48, 80, 125, 180, 260, and 360.

This instrument is denoted by E below. In 1870 and 1871, before this instrument was erected, he used Mr Gladstone's $7\frac{1}{2}$-inch achromatic.

5. *Comparison Stars.*—The brighter comparison stars *a*, *b*, *c*, etc., are the same as those of Pogson and Knott, except that Knott's star *l* was at first denoted *x* by Baxendell. As, however, there is no doubt about the identification, and as he himself used *l* after 1864, I have (though with some misgivings) altered the notebook *x* into *l* in what follows. He made one or two determinations of the magnitudes of the brighter stars, but not many: probably he accepted these data from Pogson (or Knott?), and no discussion of his separate observations is necessary. But see the letter from Father Hagen, appended to this paper.

6. As regards faint stars, Baxendell made some noteworthy observations in the early years. One instance does not affect the observations of the variable, but may be given here as an indication of what he could see. He makes the following notes (I have written ξ and η for *x* and *y* to avoid confusion with another *x* and *y*):—

"1858 February 1. Pogson's star *c* is a delicate triple, and there is a fourth star perhaps sufficiently near to constitute it a quadruple; ξ about $13\frac{1}{4}$ mag.; η about 14 mag., and of a decided light blue colour."

"1858 February 8. Companions of Pogson's star *c*, ξ about $13\frac{1}{4}$ mag., η about 14 mag."

"1858 April 17. Pogson's star *c* quadruple; A 9·2 mag., B 12, C 13½, D 14; C a light *blue* colour, very striking for so small a star. The magnitudes merely estimated."

There are no other references to these stars except that a diagram is appended on February 1, though without any scale.

7. Now there is some confusion about Pogson's *b* and *c* (see *M. N.*, lxvii. pp. 125–8), Knott using *c* and *b*, where Pogson used apparently *b* and *c*. Assuming Baxendell's identification to be the same as Knott's, *c* is the double star Σ1158; and the position angle and apparent magnitudes given in Mr Lewis's recent catalogue of the Struve stars accord well with the diagram in Baxendell's notebook for the pair marked *c*. This supplies the scale of the diagram approximately, and it can be accordingly read as follows (very roughly):—

			Mags.	
	Pos. Angle.	Dist.	Bax.	Σ.
A	0	0″·0	9·2	8·8
Σ Companion or B	(333°)	(7″·5)	12	10·0
η or C	230	9″	13¼	
ξ or D	320	60″	14	

But there is no mention of either ξ or η in *Memoirs R.A.S.*, vol. lvi. p. 226. Perhaps some double star observer may care to look at Σ1158 again. (See note by Mr Lewis at end.)

8. The faint stars in the neighbourhood of the variable, used in comparisons, are given in a diagram on 1858 February 1, which includes Knott's *f*, *g*, *h*, and *l*, so that there is no difficulty in reading it. In the neighbourhood of U there are no less than *five* other stars, two in ink, and three in pencil perhaps added afterwards. One of these is lettered as the variable U, but this identification can only be taken as provisional, as subsequent notes show. The best plan seems to be to give the positions of these stars as below (so that anyone can make an accurate diagram for himself in a few moments if he so desires), and the relevant notes just as they stand in the notebooks. For reading the diagram, the particulars given in Hagen's *Atlas Stell. Var.* have been used. A scale was constructed to fit them as nearly as possible, and the comparison of data with results shows the skill with which Baxendell made such diagrams.

TABLE I.

Knott's Letter.	Hagen's No.	Hagen's Δα	Hagen's Δδ	Baxendell's Δα	Baxendell's Δδ	Baxendell's Letter.
		m s	′	m s	′	
f	18	+0 2	+5·4	+0 0	+5·4	*f*
g	25	+0 18	−0·1	+0 18	+0·5	*g*
h	24	+0 6	+4·4	+0 6	+4·0	*h*
	23	+0 7	−3·8	+0 9	−3·8	*k*
l	39	−0 2	+2·1	−0 1	+2·2	*l*
k	33	+0 4	−2·9	+0 5	−1·5	*w*
				0 0	+0·3	U ; ink.
				+0 2	+0·1	Pencil.
				+0 2	−0·3	Ink.
				+0 4	−0·2	Pencil.
				+0 4	+1·0	Pencil.

9. The notes are as follows :—

1858 February 1. 9^h to $9\frac{3}{4}^h$. Mr Worthington's 13-inch reflector.

h 1·3 > *l* ·4 or ·5 > U Gem.
h ·7 > *w* ·7 or ·8 > U ·3 to ·5 > min. vis.

I believe the star which I have marked U will prove to be the variable, now on its march to another maximum; though very small it is distinctly defined, has no haziness about it, and has a dull yellow colour. The above estimations make U = 14·0 mag., *w* = 13·17, *l* = 13·62, and the vanishing mag. with 13 inches aperture = 14·4 to 14·5.

1858 February 4. 7 to $9\frac{1}{2}$ *h*. 13-inch reflector.
h ·7 or ·8 > *w* ·5 or ·6 > *l*. *h* 1·5 > *l* ·6 > U. *w* 1·0 or 1·2 > U. U ·4 > a small star *y* not noticed February 1. U is therefore 14·3 mag., or ·2 or ·3 less than on the 1st inst., and is perhaps therefore not the variable. *y*, which *is* perhaps the variable, is 14·7 mag.

1858 February 6. 7^h to 8^h. Mr Williamson's 12-inch reflector.
l about ·4 > *min. visible.* Supposed U and *y* occasionally seen by transient glimpses, and *y* is perhaps the brightest (say both under 14·2 mag.).

1858 February 7. 8^h to $9\frac{1}{2}^h$. 13-inch reflector.
h ·8 > *w* ·7 > *l* ·8 > supposed U (say *u*) ·2 > *y f* ·4 > *k* ·4 > *g*. ∴ *u* = 14·5 or 14·6 mag., and *y* = 14·8 mag.

1858 February 8. 8^h to $9\frac{1}{2}^h$. 13-inch reflector.
l ·6 > *u* ·3 > *y* a glimpse star near to and following *y*. *u* = 14·4 and *y* = 14·7 mag.

1858 February 17. 9^h to 10^h. Mr Williamson's 12-inch reflector.

h 1·2>w ·3>l ·6>u ·2 or ·3>y. w still appears to be decreasing. The light of u very changeable, sometimes appearing fully equal to l, and at other times quite disappearing. h 1·5>l.

Say $u=14{\cdot}5$ $y=14{\cdot}7$ mag.

1858 February 18. $8\frac{1}{2}^h$ to $9\frac{1}{2}^h$. 13-inch reflector.

w ·4 or ·5>l ·6>u ·3>y. h 1·0>w.

Say $u=14{\cdot}5$. $y=14{\cdot}8$ mag.

u precedes g $19^s{\cdot}0$ and on same parallel; or, if any difference, u south, but not to the extent of 5″; observation difficult, as u will not bear an illumination sufficient to render the spider lines distinctly visible.

1858 February 19. 9^h to 10^h. 13-inch reflector.

h 1· or 1·2>w ·4>l ·5>u ·2 or ·3>y.

1858 February 20. $8\frac{1}{2}^h$ to $9\frac{1}{2}^h$. 13-inch reflector.

w ·4>l ·5>u ·2 or ·3>y, suspicion of a glimpse star near to and sf star u. Fluctuations in the brightness of u, which I have frequently observed lately, are very striking to-night.

1858 February 22. 8^h to 9^h. 13-inch reflector. w and l both seen occasionally pretty steadily; u seen two or three times by glimpses, but no change since last observed. Moon too near for a very satisfactory observation.

1858 February 25. 9^h to $9\frac{1}{2}^h$. 13-inch reflector. Owing to the strong moonlight, I can only say that U Gem. is below 13 mag., star w glimpsed occasionally.

1858 February 28. $8\frac{1}{2}^h$. 13-inch reflector. w and l both occasionally seen, but owing to the moonlight, etc.

1858 March 6. 8^h to 10^h. 13-inch reflector.

h ·1>g ·9 or 1·0>w ·3>l ·4 or ·5>u ·2>y. Fluctuations in brightness of u noticed again to-night.

1858 March 13. $9\frac{1}{4}^h$. 7-inch reflector, p. 130.

w and l occasionally seen pretty steadily by glimpses, and even u two or three times seen in transient glimpses, or u and y, as in the best glimpses u had the *cometary* appearance of a very faint coarse double star. The difference of mag. between w and l cannot be more than ·2.

1858 March 16. 8^h to 10^h. 13-inch reflector, p. 196.

g=or ·1>h: f 1·0 to 1·2>h: w ·4>l ·4 or ·5>u ·2>y. Occasionally a suspicion of a small companion sf u about 15″ distant.

1858 May 1. $9^h\ 25^m$. 13-inch reflector, p. 199.
$u \cdot 2 > l$. When u first became visible in the twilight l was quite invisible. $9^h\ 40^m$ $u \cdot 2 > l$. 10^h (after interruption by clouds) $l =$ or $> u$: $10^h\ 10^m$ $l \cdot 2 > u$: $w \cdot 6 > u$.

1858 May 3. $9\frac{1}{2}^h$ to $10\frac{1}{2}^h$. 13-inch reflector, p. 199.
$w \cdot 4 > l$ $\cdot 4 > u$ $\cdot 3 > y$. No indications of any minute star near u, although carefully looked for.

1858 May 4. $9\frac{1}{2}^h$ to $10\frac{1}{2}^h$. 13-inch reflector, pp. 199 and 350.
$w \cdot 4 > l$ $\cdot 4$ or $\cdot 5 > u$ $\cdot 3$ or $\cdot 4 > y$. Could not satisfy myself of the existence of any minute star near u.

1858 November 12. 12^h. 7-inch reflector, p. 130.
l glimpsed; two or three transient glimpses of u. ?U 14 mag.

1858 November 14. 12^h. 7-inch reflector, p. 40.
U Gem. has suddenly burst forth, and is now = Pogson's $b = 9{\cdot}3$ mag. With power 130 it is much less sharply defined than a, b, or c, and compared with them has somewhat of a nebulous appearance. Its colour is *white*. It appears to be exactly on the place of my star u.

1859 February 27. 9^h. 13-inch reflector, p. 199.
$f \cdot 8 >$ U $\cdot 2 > h$ U 12·2 mag. Neither with achromatic p. 223 nor reflector pp. 199 and 300 is U so well defined as the neighbouring stars; but I think it is not so hazy-looking as it was in November last. y only seen by glimpses, and no star seen or suspected near U.

1859 February 28. 13^h. 13-inch reflector, pp. 199 and 300.
U $\cdot 2 > w$ $h \cdot 8 >$ U $\therefore$ U 13·1 mag. No small star visible very near U, and from the position of U with respect to w, x, and y, I can have no doubt that U and Winnecke's star are identical. U *white* or *bluish white*; light steady and haziness still perceptible.

1859 April 22. $9\frac{3}{4}^h$. 13-inch reflector, p. 196.
$l \cdot 4$ or $\cdot 5 > u$ $\cdot 3$ or $\cdot 4 > y$. From the position of u with respect to w, l, and y, I have no doubt it is the variable. It has, however, a ruddy appearance, and is, when best seen, very sharply defined.

1859 November 30. 13-inch reflector, pp. 81 and 196.
u and y both seen. U below 14·2 mag.

This last note is interpreted to mean, "if U is not the same as u, it is below 14·2 mag.," for it seems clear that Baxendell considered u to be the variable.

10. These are practically all the notes that help in any way to identify the small stars. When, as on 1858 February 28, moonlight or haze interfered, the note has been curtailed, and generally notes that do not help us have been omitted. The comparisons of

these small stars are collected below in Table II. It will be seen that Baxendell observed sometimes to 0·05, taking his unit as approximately 0·1 magnitude, which by comparison with Hagen's magnitudes we see is very nearly correct. Adjusting the differences to fit the four stars *g*, *h*, *w*, *l* as nearly as possible, we have—

	25=*g*	24=*h*	33=*w*	39=*l*	*u*	*y*
Hagen . . .	11·1	11·0	12·1	12·6	...	...
Baxendell . .	11·05	11·10	12·05	12·45	12·90	13·20

And we may compare with these Baxendell's own absolute determinations for *u* and *y*, viz. *u* = 14·5 (four nights), *y* = 14·8 (four nights); showing that his adopted magnitudes are in excess, as is indicated above in the case of Σ1158.

TABLE II.

Baxendell's Comparisons of Faint Stars.

Date.	*g*	*h*	*w*	*l*	*u*	*y*
1858 Feb. 1	...	7	...	4½	...	
4	...	7½	5½	6	4	
7	...	8	7	8	2	
8	...	...	...	6	3	
17	...	12	3	6	2½	
18	...	..	4½	6	3	
19	...	11	4	5	2½	
20	...	...	4	5	2½	
Mar. 6	−1	11	3	4½	2	
13	−1	...	2	...	...	
16	+½	...	4	4½	2	
Apr. 1	+1	10	2½	4½	4	
4	+2	9	4	4	5	
6	+1	...	4½	3	5½	
10	+1	...	4½	3½	...	
13	...	...	4½	2½	...	
17	...	...	3	4	...	
May 3	...	...	4	4	3	
4	...	...	4	4½	3½	
5	...	...	4	4	...	
6	...	...	4	4	...	
10	...	...	4	3½	3½	
11	...	...	4	4	...	
1859 Apr. 22	...	...	...	4½	3½	
Mean	+·05	+·95	+·40	+·45	+·30	

11. *The Observations of the Variable.*—The observations are collected in Table III. in a concise form. Baxendell used the symbol > in his notebooks; but as it can always be inferred from the order of the other symbols, it has been omitted. When he gives alternative readings such as ·5 or ·6, the mean has been taken. Finally the decimal points have been omitted. Thus his record

$$h \cdot 7 \text{ or } \cdot 8 > w \cdot 5 \text{ or } \cdot 6 > l$$

becomes

$$h\ 7\tfrac{1}{2}\ w\ 5\tfrac{1}{2}\ l.$$

Other contractions will be readily understood. Thus on 1859 May 20 the inference $< b + 15$ means that the faintest star visible was judged to be a magnitude and a half fainter than *b*.

The letter *u* has been retained where Baxendell used it, although he afterwards decided that it was the variable near minimum. It seems advisable to indicate as closely as possible the gradual exploring of the small stars by keeping the notebook records.

12. The early observations are difficult to give in tabular form, and are better simply transcribed as follows:—

1857 February 25. Mr W.'s 5 in. ach. p. 68. Several very small stars about the place of U Gem.; but all much less than the star to the south, about $9\frac{1}{2}$ mag.

1857 March 4. 10^h. With 5 in. achr. Stars not brighter than 11 mag. in the place of U Gem.

1857 March 8. 11^h. With 5 in. ach. p. 68. A star of about 11 mag. in the place of U Gem., and another near it nf 2′ or 3′ distant and about half a mag. less.

1857 March 16. About 8^h. 5 in. ach. p. 68. No star brighter than 11 mag. about the place of U Gem.

1857 March 21. $11\frac{1}{2}^h$. 30 in. ach. Nothing visible in the place of U Gem.

[1857 March 24 is the last date in the book preceding April 13.]

1857 April 13. 30 in. ach. A very minute star occasionally visible about the place of U Gem., and therefore about $10\frac{1}{4}$ mag.

[1857 April 14, 15, 16. Other stars observed, but U Gem. apparently not looked for.]

[1857 April 18. $9\frac{1}{2}^h$. 5 in. ach. A diagram is made of the stars *f*, *h*, *g* and three brighter (Hagen's 18, 24, 25, 22, 21, 19), and the remark made "small companion of *f*" (that is $h = 11{\cdot}0$) "almost a *min. visible*, and therefore about $12\frac{1}{2}$ mag." Baxendell's magnitudes are too large, as shown elsewhere.]

1857 April 19. 10^h. 30 in. ach. Nothing visible on the place of U. Gem.

[1857 April 20. 10^h. 5 in. ach. p. 68. Sketch of April 18 referred to, and Hagen's 18 judged ·1 or ·2 brighter than 21.]

13. There is nothing more before October, and the observations from this point may be given in tabular form.

TABLE III.

Date.			Telescope and Power.	Observations and Inferences.
1857.	d	h h		
Oct.	28		T 35	Not seen.
Nov.	18		T 35	,, <11·0.
	23		,,	,,
	26	11½	,,	,,
1858.				
Jan.	20	9½	A 68	,,
	21	13	T 35	,,
	25	11½	T 40	,, <*b*+10.
	31	10	R(12)	,, <13·0.
Feb.	1	9½	R(13)	*l* 4½ *u* ; *w* 7½ *u* 4 limit.
	4	7–9½	R(13)	*l* 6 *u* : *w* 11 *u* 4 *y*.
	6	7–8	R(12)	*u*=*y*.
	7	8–9½	R(13)	*l* 8 *u* 2 *y*.
	8	8–9½	R(13)	*l* 6 *u* 3 *y*.
	17	9–10	R(12)	*l* 6 *u* 2½ *y*.
	18	8½–9½	R(13)	*l* 6 *u* 3 *y*.
	19	9–10	R(13)	*l* 5 *u* 2½ *y*.
	20	8½–9½	R(13)	*l* 5 *u* 2½ *y*.
	22	8–9	R(13)	Not seen <*l*.
	25	9–9½	R(13)	,, <*w*.
	26	8½	R(13)	,, <*w*.
	28	8½	R(13)	,, <*l*.
Mar.	3	9	T 35	,, <*g*.
	6	8–10	R(13)	*l* 4½ *u* 2 *y*.
	11	10	A 68	Not seen <*l*.
	13	9¼	R(7) 130	,, <*l*.
	14	8	A 68	,, <*w*.
	16	8–10	R(13)	*l* 4½ *u* 2 *y*.
	20	8–10	A 40	Not seen <*g*.
	24	8	R(13) 81	,, <12·6.
	26	8¼	R(13) 81	,, <*g*.
Apr.	1	8–10	R(13) 196	*l* 4½ *u* 4 *y*.
	4	8½–9½	R(13) 196	*l* 4 *u* 5 *y*.
	6	8½–9½	R(13) 196	*l* 3 *u* 5½ *y*.
	9	9½	R(7) 40	Not seen.
	10	8½–10	R(13) 196	*l* 3½ *u* 4 limit.

Date.			Telescope and Power.	Observations and Inferences.
1858.	d	h h		
Apr.	12	9½	A 40	Not seen <*w*.
	13	8½–9½	R(13) 196	*l* 2½ *u*.
	17	9–11	R(13) 199	*l* 4 *u* 3 limit.
	21	9¾	R(13) 196	Not seen <*w*.
	23	9½	R(13) 196	,, <*l*.
	25	9	R(13) 196	Not seen <13.
May	1	9½	R(13) 199	See above notes.
	3	9½–10½	R(13) 350	*l* 4 *u* 3 *y*.
	4	9½–10½	R(13) 199	*l* 4½ *u* 3½ *y*.
	5	9¾–10½	R(13) 199	*l* 4 *u*.
	6	10–10½	R(13) 199	*l* 4 *u*.
	10	10–10½	R(13) 199	*l* 3½ *u* 3½ *y*.
	11	10–11	R(13) 199	*l* 4 *u*.
	16	10–10½	R(13) 199	Not seen <*w*.
	18	10–10½	R(13) 199	,, <12·8.
Sept.	20	15	R(13) 196	*u*=*l*.
Nov.	2	...	(22)*a*	Not seen <*c*+5.
	5	...	(22)*a*. 21	,, <10·2.
	6	...	(22)*a*. 21	,, <10·5.
	10	12½	R(7) 130	,, <*l*.
	11	12½	R(7) 130	,, <*l*.
	12	12½	R(7) 130	*u* glimpsed.
	15	12	R(7) 40	U=*b*.
	16	11	R(7) 40	*b* 1½ U.
	17	11–11¾	R(7) 40 / 130	*b* 5½ U 13½ *f*.
	18	13	R(13) 81	*b* 12 U 2½ *e*.
	21	12½	R(13) 199	Not seen <*w*.
Dec.	2	9½	R(13) 199	,, <*l*.
1859.				
Feb.	22	10½	A 68	*a* 2½ U 2½ *b*.
	23	12¾	(22)*a* 21	*c* 2 U : *b* 1 U.
	24	7	A 68	*c*=U.
	26	12⅓	(22)*a*	*e* 2 U.
	27	7	A 68	*f* 7 U 3 *h*.
	27	9	R(13) 199	*f* 8 U 2 *h*.
	28	13	R(13) 300	*h* 8 U 2 *w*.

TABLE III.—*continued.*

Date.			Telescope and Power.	Observations and Inferences.
1859.	d	h m		
Mar.	10	8½	R(13) 300	*y* seen.
Apr.	22	9¾	R(13) 196	*x* 4½ *u* 3½ *y*.
May	13	10½	A 40	Not seen.
	16	10	A 40	,, <11·0.
	20	10	A 40	,, <*b*+15.
	22	10½	A 40	,, <*b*+15.
	23	10½	A 40	,, <10·7.
Nov.	30	...	R(13) 196	*u* and *y* seen.
Dec.	3	...	A 68	Not seen <10.
	5	...	R(13) 196	,, <*l*.
	14	...	R(13) 196	*u* and *y* seen.
1860.				
Feb.	16	10	R(13) 199	*l* 5 U.
	26	9	A 68	Not seen <*w*+.
	28	9¼	A 68	,, <13·8.
Mar.	2	8	A 223	,, <11·0.
	8	8	A 223	,, <13·0.
	14	10	R(13) 199	U=14·3.
	18	9¼	A 39	Not seen <10·3.
	21	9	A 223	,, <13·7.
	25	10	R(13)	,, <14·0.
Apr.	10	8¾	A 68	,, <13·0.
	15	10	A 68	,, <13·0.
	20	9½	A 68	*a* 2 U.
	21	9	A 68	*a* 3½ U 3 *c*.
	22	9	A 68	*a* 2 U 4½ *c*.
	23	9	A 68	*a* 6 U 1 *c* : U 1½ *b*.
	24	9¼	A 68	*c* 1½ U : *b* ½ U.
	25	9	A 68	*c* 1½ U : *b* ½ U.
	27	9	A 68	*b* 2 U : *a* 10 U.
May	1	10½	...	*f* 4 U.
	3	9½	R(13) 196	*g* 10 U : *h* 7½ U 2½ *w*.
1861.				
Jan.	1	10 50	C 39	Not seen <10·0.
	3	13 +	R(7) 39	,, <12·3.
	4	9 +	A 68	,, <12·3.

Date.			Telescope and Power.	Observations and Inferences.
1861.	d	h m		
Jan.	6	9 5	A 68	Not seen <13·0.
	15	10 10	A 68	,, <13·3.
	28	9 35	A 68	,, <11·0.
Feb.	6	12 25	C 39	,, <12·0.
	10	9 5	A 68	,, <13·3.
	24	7 40	A 68	,, <11·0.
	28	10 25	C 39	,, <11·0.
Mar.	8	10 15	C 39	,, <10·5.
	13	8 35	R(13) 81	*l* 5 U.
	16	10 5	A 68	Not seen <13·3.
	21	8 30	A 68	,, <9·5.
	30	8 35	A 68	,, <13·3.
Apr.	3	9 10	A 68	,, <13·3.
	6	8 30	A 68	,, <12·5.
	9	10 40	C 39	,, <10·5.
	10	10 30	R(13) 196	*l* 4 U.
	11	8 50	A 68	Not seen <13·3.
	14	9 30	R(13) 81	*l* 4 U.
	20	8 30	A 68	Not seen <11·0.
	28	10 0	A 68	,, <13·3.
May	4	10 0	A 68	*a* 2 U 3 *c*; U 3½ *b*.
	9	9 30	A 68	*b* 2 U ; *c* 2 U.
	13	9 30	A 68	Not seen <10·5.
	14	10 25	A 68	U 4 *f*.
Oct.	28	11 40	,,	Not seen <12·5.
Dec.	28	9 0	,,	*w* 3½ U = *l*.
1862.				
Jan.	5	9 15	,,	*a* 8 U: *b* 1 U 1 *c*.
	10	9 30	,,	? visible <10·2
	11	8 15	,,	*f* = U.
	22	8 15	,,	Not seen <12·5.
Mar.	15	...	,,	,, <11·0.
	19	10 0	,,	,, <13·3.
	22	9 0	,,	,, <11·0.
	31	10 5	,,	,, <12·0.
Apr.	8	10 30	,,	,, <10·5.
	10	8 25	,,	,, <11·5.

TABLE III.—*continued.*

Date.			Telescope and Power.	Observations and Inferences.
1862.	d	h m		
Apr.	11	8 50	A 68	Not seen < 10·5.
	12	...	,,	Suspected < 11·0.
	13	...	,,	Not seen < 12·0.
	23	9 15	,,	Glimpsed < 13·1.
	28	10 0	,,	?glimpsed < 13·5.
	29	10 30	,,	Not seen < 13·3.
	30	9 45	,,	Not seen < 13·3.
May	3	9 45	R(13) 81	Barely glimpsed < 13·3.
	5	10 30	A 68	Not seen < 12·5.
	16	10 10	A 68	,, < 11·0.
Oct.	18	12 50	C 39	,, < 10·5.
Nov.	15	13 10	,,	,, < 10·0.
	16	10 10	R(13) 81	,, < 13·5.
	23	...	A 68	,, < 13·0.
Dec.	26	9 0	,,	$e\frac{1}{2}$ U 6 *f*.
	29	11 30	,,	Not seen < 12·2.
	30	14 0	,,	,, < 12·2.
1863.				
Jan.	23	7 40	,,	,, < 13·3.
Feb.	8	7 50	,,	Glimpsed ? < 13·3.
	12	11 0	,,	Not seen < 13·3.
	14	9 0	C 39	,, < 12·0.
	15	9 0	A 68	,, < 13·3.
	17	...	B 88	,, < 14·0.
	21	8 10	A 68	,, < 13·7.
Mar.	3	9 40	,,	,, < 11·0.
	15	11 0	,,	,, < 13·3.
	21	11 0	,,	,, < 13·0.
	23	9 0	,,	,, < 13·3.
	26	8 45	,,	,, < 13·0.
Apr.	1	9 30	,,	,, < 10·0.
	11	8 40	,,	*b* 2 U $6\frac{1}{2}$ *d*: U 14 *e c* 3 U.
	13	9 0	,,	*e* $2\frac{1}{2}$ U 1 *f*: *d* $8\frac{1}{2}$ U.
1863.				
Apr.	15	9 0	A 68	Not seen < 13·0.
Nov.	2	12 0	,,	,, < 11·0.
	4	12 20	C 39	,, < 10·5.
	5	12 40	,,	,, < 11·5.
	8	13 30	,,	,, < 10·5.
Dec.	2	13 0	,,	,, < 10·5.
	5	10 15	A 68	,, < 12·0.
	17	8 30	,,	,, < 10·5.
	27	7 30	,,	*b* 4 U.
	29	9 30	C 39	Not seen < 10·7.
1864.				
Jan.	4	8 30	A 68	,, < 13·3.
Feb.	12	...	,,	,, < 13·0.
Mar.	15	9 10	,,	,, < 10·9.
	23	9 40	,,	,, < 12·0.
	28	9 40	,,	,, < 13·3.
Apr.	11	9 0	,,	,, < 13·3.
	13	...	,,	,, < 11·5.
	18	...	,,	,, < 11·5.
	21	...	,,	,, < 12·3.
	25	...	,,	,, < 13·0.
	28	...	,,	Suspected < 13·0.
May	7	10 10	,,	Not seen < 13·2.
Oct.	7	..	R(13)	Glimpsed < 13·7.
	31	..	A 68	Not seen < 12·5.
Nov.	5	...	R(13)	Doubtful < 13·5.
	28	...	A 68	Not seen < 13·3.
Dec.	21	...	,,	,, < 13·0.
	30	7 40	,,	*b* 1 U = *c*.
	31	11 10	,,	*a* 6 U 3 *b* = *c*.
1865.				
Jan.	4	8 20	,,	*a* 12 U = *c* : *b* $1\frac{1}{2}$ U.
	6	...	,,	*c* $1\frac{1}{2}$ U.
	20	...	,,	Not seen < 13.
Mar.	30	...	,,	,, < 13.
Apr.	10	...	,,	,, < 10·5.
	15	...	,,	,, < 12·5.

TABLE III.—*continued.*

Date.	Telescope and Power.	Observations and Inferences.
1865. d h m		
Apr. 24 10 0	A 68	f 9 U 3 h : U 1 g 12·1.
25 10 0	,,	g 4 U 12·6.
26 10 0	,,	k 1 U 13·4.
Nov. 21 ...	,,	Not seen < 13·3.
Dec. 14 ...	,,	,, <13·3.
22 ...	,,	,, <12·5.
26 ...	,,	,, <12·3.
1866.		
Jan. 15 9 40	,,	,, <13·3.
16 10 25	,,	h 6½ U 5 k, g 4 U 12·9.
18 13 30	C 39	a 6 U 2 c ; U 3 b.
20 8 15	A 68	a 6½ U 2 b ; U 1½ c.
22 7 10	,,	c 1 U, b 1½ U.
23 11 20	,,	a 6 U 1½ b ; U 2 c.
28 ...	,,	Not seen < 10·3.
29 ...	,,	,, <10·5.
Feb. 2 8 40	,,	k 4½ U ; l = U 13·7.
Mar. 13 ...	,,	Not seen < 13·5.
Apr. 13 ...	,,	,, <13·3.
21 9 50	,,	f 3 U 8 h.
22 10 30	,,	h 4½ U.
Oct. 9 ...	,,	Not seen < 12½.
1867.		
Jan. 2 ...	,,	,, <13·3.
8 ...	,,	Suspected < 13·3.
11 ...	,,	Not seen < 13·3.
16 ...	,,	,, <10·2.
28 ...	,,	,, <13·4.
Feb. 2 ..	,,	,, <13·2.
20 ...	,,	,, <13·0.
26 ...	,,	,, <13·0.
Apr. 21 ...	,,	,, <13·0.
May 5 ...	C 39	,, <10·5.
14 ...	A 68	,, <12·0.
1867. d h m		
Nov. 17 ...	A 68	Not seen < 13·0.
Dec. 18 8 10	,,	U 2 f.
22 8 50	,,	Not seen < 12·8.
1868.		
Apr. 11 ...	,,	,, <13·3.
23 ...	,,	,, <13·2.
1869.		
Feb. 5 ...	,,	,, <12·8.
Mar. 2 ...	,,	,, <13·5.
Dec. 23 ...	,,	,, <12·5.
1870.		
Mar. 25 ...	G 60	l 3½ U.
1871.		
Mar. 13 ...	C 39	Not seen.
Apr. 7 ...	G	,, <14·0.
10 ...	,,	,, <13·7.
1877.		
Dec. 12 ...	E 80 & 180	Not seen < 13·5.
1878.		
Jan. 9 ...	...	,, <13·0.
Mar. 24 ...	E 80	,, <13·5.
31 ...	,,	,, <13·5.
April 1 ...	,,	,, <13·5.
5 ...	,,	Glimpsed 13·7.
6 ...	,,	Barely glimpsed.
8 ...	,,	Not seen < 12·5.
17 ...	,,	,, <13·0.
May 1 9 40	,,	f 6 U 4 h ; U 7 g.
3 10 30	,,	Not seen < 13.
Nov. 2 11 50	,,	,, <13·5.
4 5 15	,,	,, <13·5.
25 ...	,,	,, <13·0.
28 ...	,,	,, <13·5.
Dec. 7 ...	,,	,, <12·5.
23 ...	,,	,,
30 ...	,,	,, <12·5.

TABLE III.—*continued.*

Date.			Telescope and Power.	Observations and Inferences.
1879.	d	h m		
Jan.	13	7 40	E 80	b 2 U : c 1 U ; a 10 U.
		8 0	,,	b 1 U = c.
	15	6 50	,,	b 1½ U ; c 1 U a 10½ U.
		8 15	,,	b 1½ U ; c 1 U.
	16	8 20	,,	b 3 U ; c 2 U 13 e.
		8 35	...	b 2½ U ; c 1½ U ;
		9 45		a 14 U 10 d.
	19	7 10	...	U ½ e ; U 7½ f.
	22	8 30	E 80	h 7 U 3 k.
		8 40	E 125	h 8 U 4 k.
	23	9 45	E 80	k 3 U 2 l.
	25	10 30	...	Not seen < 13·0.
April	10	9 15	..	Not seen < l + 2.
Sept.	19	14 30	...	,, < k + 3.
Oct.	14	12 50	...	,, < 13·5.
	15	13 0	...	,, < k + 3.
	20	13 0	...	,, < k.
Nov.	1	12 0	...	,, < h + 7.
1880.				
Jan.	18	12 0	...	Glimpsed = 13·5
	19	11 0	...	Not seen < k + 3.
	22	12 15	...	,, < 13·0.
	28	7 0	...	a 9 U = b 1½ c.
	29	10 40	...	b = U a 8 U.
	31	12 20	...	b 2½ U 12½ e ; c 1 U 10½ d.
Feb.	1	9 30	...	b 1½ U ; c ½ U.
	3	7 10	...	b 4 U 5 d ; U 10 e.
	5	9 25	...	d 2 U 5 e.
	8	9 0	...	h ½ U : g 7½ U.
	14	...	...	Not seen < k.
Apr.	29	10 0	...	,, < 13·0.
	30	11 0	...	Glimpsed 13·7.
May	1	9 50	...	Not seen.
	3	10 45	...	Glimpsed : k 2½ U.

Date.			Observations and Inferences.
1880.	d	h m	
May	6	10 20	Not seen < 13·0.
	9	...	,, < g.
	10	10 40	,, < k.
	12	10 35	,, < 13·5.
Oct.	9	12 40	,, < k.
	10	13 20	,, < k.
Nov.	26	11 30	k 5 U ; l 2½ U.
	27	12 42	= 13·7.
Dec.	1	10 15	Not seen < k.
	24	9 20	f 10 U 1½ g ; U 4½ h.
	25	8 25	U 2 k ; U 5 l.
1881.			
Apr.	1	8 50	f 10 U 2 h 1 g.
		11 55	U 1½ f : U 12½ h.
		12 14	U 4 f ; U 3 e ; d 2 U 15 h.
		12 35	U 5 f : d 1½ U 3 e.
	2	8 15	U 1 b : a 5 U.
	3	9 5	a 5½ U 1½ b : U ½ c.
	5	8 40	b 1 U ; c 2 U : a 7 U.
	6	10 40	b 1½ U ; c 2½ U ; a 7 U 12 d.
	7	9 5	b 1 U : c 2 U.
	8	8 35	b 4 U : c 5 U a 10 U 10 d.
	9	9 15	b 7 U 8 d : a 13 U : c 8 U.
	17	9 10	Not seen < 13·8.
Oct.	26	...	,, < k.
Nov.	14	...	,, < 13·5.
	17	...	,, < l.
Dec.	27	...	,, < k.
1882.			
Mar.	6	8 15	f 5 U 4 g : U 8 h.
	8	11 20	Not seen < h.
	12	12 0	Glimpsed < l.
Oct.	28	12 0	Not seen < k.
1883.			
Jan.	30	7 45	a 5 U = b.
1884.			
Jan.	20	...	Not seen < k.

TABLE III.—*continued.*

Date.			Observations and Inferences.
1884.	d	h m	
Feb.	2	...	Not seen $< k+2$.
May	18	10 30	c 12 U 5 e : d 3 U.
	19	10 20	d 6 U 4½ e : c 17 U.
Dec.	20	12 0	l 3 U.
1885.			
Jan.	5	10 30	e 4½ U 2½ f.
	6	7 35	f 12 U 2 h ; U 3 g.
	7	10 5	h 6 U 7 l : g 5 U 5 k.
	8	...	k 3 U = l.
Mar.	22	...	Not seen $< k$.
Apr.	2	...	,, $< l$.
	8	8 40	a 7 U = b ; U 1½ c.
Nov.	15	11 0	Not seen $< 12{\cdot}5$.
Dec.	1	10 0	,, $< 13{\cdot}0$.
1886.			
Jan.	4	...	,, $< 13{\cdot}5$.
	5	10 50	,, $< 13{\cdot}7$.
	29	...	,, $< k$.
	30	...	,, $< l$.

Date.			Observations and Inferences.
1886.	d	h m	
Jan.	31	...	Not seen $< l$.
Feb.	14	...	,, $< g$; h 5 g.
	23	...	,,
Mar.	3	11 5	k 2 U 2 l.
	6	8 0	b 3½ U ; c 2 U 8 d.
	7	8 30	b 4 U ; c 2 U.
	8	7 55	b 2½ U ; c ½ U 8½ d.
	9	7 50	b 1½ U = c ; U 10 d.
	10	7 45	b 3 U ; c 1 U 10 d.
	11	7 35	b 4½ U : c 3½ U 8 d.
	12	7 50	b 6 U : c 5 U 7 d.
Nov.	30	10 5	b = U 1½ c.
Dec.	1	11 20	b = U 1½ c.
	4	9 5	c 4 U 7 d.
	4	10 15	c 3½ U 7 d ; g 5 h.
1887.			
Feb.	26	8 0	about = f.
	27	8 0	f 7 U 3 h.
	28	9 30	h 10 U 7 k : f 8 h 1 g.

[The only remaining entries in the book are on March 3 (three stars), June 7 (one star), June 10 (five stars). From 1879 April there are no special entries about the telescope, which may be assumed to be E throughout.]

The following letter from Father Hagen explains itself. He draws attention to an unlucky misprint in my paper on p. 128, viz. Pogson's number for m in the first column should be, not 113, but 115, as in the diagram. After some consideration I have assumed that Baxendell's k was not Knott's k, but this star m (Hagen's 23): and Baxendell must have used w for Knott's k (Hagen's 33). The identification of these faint close stars is rather troublesome.

Letter from Father Hagen on the Comparison Stars for U Geminorum.

My Dear Professor Turner,—Your publication of Pogson's observations of *U Geminorum* in the *Monthly Notices*, vol. lxvii. pp. 119–131, will prove very useful to students interested in this line of astronomy. The publication might perhaps be enhanced in its value by a few statements regarding Pogson's comparison stars. In making a final discussion of Pogson's observations, one will naturally not be satisfied with the identification of the comparison stars from diagrams alone, or with the preliminary magnitudes assigned to them by Pogson and Knott.

The exact identification of all the stars of his chart was made by Pogson himself in a manuscript catalogue now preserved at the Harvard College Observatory. The catalogue was made within the years 1856 to 1860 at the Radcliffe and Hartwell Observatories. On five pages it gives the R.A. and Decl. of all the stars on the chart, reduced to 1860·0. A complete illustration of Pogson's catalogues may be seen in a publication of Georgetown College Observatory, entitled "Supplementary Notes," etc., pp. 28–31. There is no chart of this Variable among the manuscripts preserved at Harvard. It is, however, not necessary to consult these manuscripts, since all of Pogson's comparison stars of *U Geminorum* can be identified by means of chart 2815 of the *Atlas Stellarum Variabilium* (Series II.).

As to the magnitudes of the stars, those of Pogson and Knott have apparently a common scale, and seem to be based on the theoretical limit of visibility in their instruments. The table subjoined gives, in addition, the magnitudes of the fainter stars by Winnecke, and those of all the stars, computed from the grades of the *A.S.V.* The latter two sets of magnitudes have again a common scale, and are both based upon the B.D. system.

The first two columns of the following table explain themselves from page 128 of your article. The third gives the letters and magnitudes of Winnecke, and is taken from the *Astr. Nachr.*, vol. xlvii. No. 1120. The two columns headed 'A.S.V. contain the numbers and magnitudes of the atlas chart 2815. From the same atlas are taken the columns $\Delta\alpha$ and $\Delta\delta$, which give the positions of the comparison stars relative to the Variable in R.A. and Decl. To them are appended two columns *D*, with the mean results deduced from your readings of the two diagrams. They may serve to show, on the one hand, how accurate the diagrams are, and, on the other, that there is no doubt left in the identifications.*

* In vol. xvii. of the *Astrophysical Journal*, p. 282, I suggested that Pogson's star 8·9 might be No. 5 of the Atlas Chart, instead of No. 1, judging from the B.D. magnitude of this star, and not having seen Pogson's diagram.

Pogson's Comparison Stars for U Geminorum.

Pogson.		Knott.		Winnecke.	A.S.V.		$\Delta\alpha$.		D.	$\Delta\delta$.	D.	Bax. [H.H.T.]
	M		M	M		M	m	s	s			
a	8·8	*a*	8·6		1	8·6	+0	4	7	−19′8	19′5	
b	9·2	*c*	9·2		3	8·8	+1	26	28	− 6·9	7·0	
c	9·4	*b*	9·3		4	9·0	+1	19	20	− 9·6	9·0	
d	10·3	*d*	10·3		9	9·6	−1	10	11	+14·5	14·5	
n	10·2				11	9·6	−0	46	46	+16·7	16	
e	10·8	*e*	10·6		16	10·0	−0	27	27	− 9·9	9·5	
f	11·3	*f*	11·2	*f* 10·9	18	10·3	+0	2	1	+ 5·4	6·0	*f*
m	11·5			*g* 10·7	23	10·8	+0	7	4	− 3·8	5	*k*
h	12·3	*h*	12·3	*e* 11·3	24	11·0	+0	6	6	+ 4·4	4·5	*h*
g	11·9	*g*	12·3	*d* 11·4	25	11·1	+0	18	18	− 0·1	+0·5	*g*
l	13·0	*k*	13·3	*b* 12·2	33	12·1	+0	4	5	− 2·9	1·5	*w*
k?	13·7	*l*	13·7	*a* 12·6	39	12·6	−0	2	5	+ 2·1	2	*l*

One word I wish to add about the remarkable fluctuation of light in this Variable as seen by Pogson on March 26, 1856. Attention has been called to this record, in substantially the same words, by Mr J. Baxendell in the *Astronomical Journal*, vol. xxii., 1902, p. 127. In the year following I had occasion to illustrate Pogson's observation of 1856 by four other instances of a similar nature, and to suggest that instantaneous fluctuations in the light of stars, when recorded by good authority, should not be rejected as unconfirmed (see *Astrophysical Journal*, vol. xvii., 1903, pp. 281–285).—Very faithfully yours, J. G. Hagen, S.J.

Specola Vaticana:
January 27, 1907.

Note by Mr Lewis.

The evening of 1907 March 11, on which I received the proof of Professor Turner's paper, being fairly good, the 28-inch refractor was set on Σ 1158 and measures made of the faint stars near. The results are :—

	Magnitudes.	Position.	Distance.
AB,	8·5 and 9·8	332°0	7″74 = Σ 1158
AC,	8·5 ,, 12·0	256·0	18·70
AD,	8·5 ,, 12·0	304·7	65·30
AE,	8·5 ,, 13·5	154·2	58·35

The measure of AB is a mean of Mr Eddington's and my own. The measures of C, D, E are by myself.—T. Lewis.

On the Classification of Long-period Variable Stars, and a possible Physical Interpretation. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

Summary.

§ 1. Reference to previous analysis of light-curves of long-period variables, and classification by A, the coefficient of sin θ in a harmonic analysis, counting θ from maximum.

§§ 2–6. Inclusion in the series of 12 curves determined by Mr J. A. Parkhurst, and formation of a set of seven typical light-curves.

§§ 7–13. The maximum calculated from the harmonic analysis does not agree with that assigned by the observer. Is the latter systematically wrong, owing to the fact that the method commonly adopted of bisecting chords is really no guide?

§§ 14–17. General sketch of physical hypothesis. If the variation in light is due to faculæ or flocculi which arise in high latitudes and approach the equator throughout most of the period, and then rather suddenly return to high latitudes, the aspect presented by the star would modify the light-curve, owing to the greater importance of the faculæ near the centre of the disc. If one of the star's poles were towards the observer, the faculæ in high latitudes would be most obvious, the equatorial faculæ being subject to foreshortening and absorption. The minimum would then follow the maximum early. Conversely, if the star's equator were towards us, it would be late.

§§ 18–21. Consideration of the actual case of the Sun, which does not give satisfactory results.

§§ 22–24. Reasons why the case of the stars may be entirely different from that of the Sun, owing to the smallness of the variation in the latter case.

§§ 25–29. Investigation of the average factor for foreshortening for different aspects of the star.

§§ 30–33. Inquiry whether foreshortening alone can explain the range in type of light-curves. The mean latitude of the faculæ must apparently change from nearly 80° to about 5°, which seems too large a range.

§§ 34–35. If we add the effect of absorption near the limb, these limits will be reduced. In default of any information as to the amount of such absorption, we can neither affirm nor deny the adequacy of the hypothesis. But a factor $\cos^2\zeta$ for the combined effect of absorption and foreshortening, instead of $\cos\zeta$ for foreshortening alone (where ζ is the distance from the centre of the visible disc), would be quite sufficient to make the range in latitude comparable with that on the Sun.

§§ 36–37. Collection of data given by Chandler for M – m, the

interval between a maximum and the preceding minimum, and comparison with the data from complete light-curves; tabulation of cases near the ends of the series, which on the above hypothesis should represent stars with (*a*) their poles turned approximately towards us, and (*b*) with their equators turned towards us.

§ 38. Remark that while class (*b*) occur in all galactic latitudes, class (*a*) are absent from the galactic poles. This suggests that the axes of long-period variables are roughly parallel to the galaxy.

§ 39. Recapitulation of the chief points of the hypothesis, and remarks on a limitation not before noticed.

§§ 40–42. Brief consideration of possible inversions of the hypothesis.

(*a*) If the faculæ quite die out at minimum.

(*b*) If the dark spots and not the bright faculæ are the main source of variation.

Neither of these seems so suitable as the original hypothesis.

§ 43. Chandler's periodic terms cannot be due to a periodic change in aspect of the star. Possibly they may arise from the coexistence of several periodicities which vary in amplitude, as Schuster has found for the Sun.

1. In *Memoirs R.A.S.*, vol. lv. p. xcviii (or see also *Monthly Notices*, vol. lxiv. p. 547), the mean light-curves for 19 long-period variables observed at the Rousdon Observatory in the years 1885–1900 are reduced to the same scale of period and of range in magnitude, and analysed harmonically so as to be closely represented by the formula

$$M + A \sin \theta + B \cos \theta + C \sin 2\theta + D \cos 2\theta + E \sin 3\theta + F \cos 3\theta$$

When the results are arranged according to the values of the coefficient A, the other coefficients are found to be in approximate sequence also.

2. When Mr Parkhurst's "Researches in Stellar Photometry, 1894–1906," * was received, containing 12 new light curves, from observations made chiefly at the Yerkes Observatory, it was natural to examine whether his curves fell into the same sequence, and the answer was sufficiently satisfactory. He has indicated in each case the epoch of maximum, and given diagrams of the smooth curves. From these curves readings were taken at twelve equidistant points, beginning with his assigned maximum. The results were then analysed harmonically, just as in the case of the Rousdon variables, and the results are given in the following table. The second decimal place must not be regarded too seriously, if only for the reason that it is not always easy to make precise readings from the curves, owing to the steep gradients which sometimes occur.

* Carnegie Institution, October 1906.

TABLE I.

(Corresponding to Table LIV., *Mem. R.A.S.*, lv. p. xcix.)

Name.	Chandler's Number.	6A	6B	6C	6D	6E	6F	Const.	Divisor.
S Cygni	7220	+0·24	−2·83	−·15	+·02	−·13	−·21	10·2	4·8
W Androm.	787	−0·36	−3·04	−·11	−·40	+·07	−·09	7·5	5·5
R U Herculis	5798	−0·54	−2·53	−·29	−·34	−·34	−·36	8·5	5·3
R Comæ	4315	−0·64	−2·84	+·20	−·56	+·08	+·02	8·8	4·9
Y Cassiop.	8629	−0·90	−2·73	−·09	−·90	+·16	−·28	9·7	4·0
V Androm.	267	−0·91	−2·67	+·25	−·14	−·12	−·12	9·3	4·2
R V Herculis	6100	−1·04	−2·73	+·53	−·03	+·08	−·08	10·5	3·9
S Lyræ	6894	−1·10	−2·55	+·64	−·28	+·02	+·13	10·2	4·6
T Androm.	103	−1·11	−2·73	−·02	−·32	+·02	−·16	8·5	4·3
[The Sun]	...	−1·49	−2·32	+·11	−·70	+·19	−·19	...	...
V Delphini	7458	−1·54	−2·24	+·06	−·64	+·04	−·16	9·7	7·0
S X Cygni	7269	−1·57	−2·58	−·14	−·20	−·02	−·36	9·1	4·2
Z Cassiop.	8518	−1·80	−2·18	+·04	−·54	−·02	−·30	10·6	4·8

3. In the series have been included the values obtained for the Sun (*Monthly Notices*, lxiv. p. 549) on the hypothesis that Wolf's spot numbers can be taken as an indication of light-variability. The spots themselves would imply loss of light; but this is probably more than compensated by the increased brilliance in the accompanying faculæ or flocculi. It is not unreasonable to compare spottedness directly with the light-curves, for the stellar magnitude of the Sun would be proportional to

$$-0{\cdot}4 \log (P+F)$$

where P is the brightness of the ordinary disc, and F the small increment due to faculæ which may be taken as roughly proportional to the observed spottedness.

Now
$$\log (P+F) = \log P + \log (1+F/P)$$
$$= \log P + kF \quad \text{approximately}$$

where k is a constant. Thus F appears in a form appropriate for comparison with stellar magnitudes, since in all cases we subtract a constant (corresponding to log P in this case) and divide by a factor (the factor k). The peculiarity of the Sun is that the subtracted constant is so large. This makes no difference at present, but it is important later (§§ 22–24).

4. A point calling for special remark is that the Sun is now included in the series instead of being outside it as before. The extreme value of A for the Rousdon variables was −1·26; but Mr Parkhurst's series contains three stars with negative values of A greater than this numerically. There is thus no longer any need to extrapolate for the Sun.

To show the accordance between the former series and that provided by Mr Parkhurst, the values of A, B, C, etc. for the Rousdon variables might have been repeated along with the above

values in Table I. But it was considered a better plan to go back to the light-curves and give the actual readings for the twelve standard points, as in Table II. The Rousdon curves are simply reproduced from Table LIII. of p. xcviii, *Mem. R.A.S.*, vol. lv.

TABLE II.

Light-Curves according to Value of A.

Name.	6A	Max.	2	3	4	5	6	7	8	9	10	11	12	
T Cassiop.	+1·23	0	16	40	63	91	99	78	48	33	32	24	10	
T Cephei	+0·75	0	16	39	63	87	102	98	78	52	38	34	12	
S Herculis	+ ·50	0	10	32	54	84	99	94	76	60	47	23	7	
S Boötis	+ ·46	0	7	22	47	76	94	100	87	57	31	14	4	
R Aurigæ	+ ·38	0	13	33	57	78	96	101	79	52	44	39	17	
S Urs. Maj.	+ ·35	0	6	19	44	67	85	98	86	53	25	14	5	
S Cygni	+ ·24	0	14	31	56	77	94	100	92	65	43	31	12	P
R Lyncis	+ ·07	0	8	26	45	66	87	100	90	62	39	27	8	
R Camelop.	+ ·07	0	6	24	46	66	86	100	88	66	40	21	7	
S Cephei	·00	0	9	27	51	75	93	98	86	64	57	35	10	
R Draco.	− ·08	0	7	27	49	75	92	101	97	80	54	24	5	
χ Cygni	− ·31	0	9	26	48	68	87	100	100	80	57	33	9	
W Androm.	− ·36	0	10	29	56	80	95	100	98	85	71	45	13	P
R U Herculis	− ·54	0	9	32	60	74	87	98	100	79	66	58	36	P
S Cassiop.	− ·63	0	10	25	45	66	87	97	97	87	65	44	16	
R Comæ	− ·64	0	6	27	53	69	82	92	100	98	84	47	10	P
R Cygni	− ·77	0	11	26	42	57	73	92	102	92	64	38	11	
T Urs. Maj.	− ·80	0	8	21	41	61	77	93	101	94	69	35	8	
T Draco.	− ·88	0	8	21	38	54	71	87	98	89	65	38	12	
Y Cassiop.	− ·90	0	12	35	62	78	90	98	100	100	92	73	18	P
V Androm.	− ·91	0	5	21	38	55	74	93	100	88	64	40	17	P
R V Herculis	−1·04	0	8	18	28	46	69	95	100	90	67	31	8	P
S Lyræ	−1·10	0	7	20	31	46	63	81	96	100	65	33	7	P
T Androm.	−1·11	0	7	21	44	63	84	98	100	96	79	56	23	P
R Urs. Maj.	−1·11	0	8	22	39	59	78	92	100	98	84	45	9	
S Coronæ	−1·15	0	5	17	32	47	62	77	91	100	73	32	4	
R Cassiop.	−1·23	0	13	30	42	47	71	93	101	95	80	58	21	
U Orionis	−1·26	0	8	24	43	63	78	92	100	99	84	62	27	
[The Sun]	−1·49	0	9	23	38	54	71	84	94	99	93	65	22	
V Delphini	−1·54	0	9	23	37	54	69	83	94	100	90	64	31	P
S X Cygni	−1·57	0	5	17	31	50	79	95	100	93	81	67	36	P
Z Cassiop.	−1·80	0	8	21	33	48	67	85	98	100	92	71	37	P

The unit for the table is 0·01, in order to avoid the use of decimal points. The curves due to Mr Parkhurst are indicated by the letter P in the last column.

5. Glancing down the columns, there are considerable irregularities, which may be too great to be ignored; but the accidental errors of light-estimations are large, errors of half a magnitude being far from impossible. [The average range in magnitude of these variables is about 4; and 0·5/4 = ·13; so that half a magnitude means 13 units in the table.]

If we may regard the errors as chiefly accidental, we may estimate mean values by grouping the stars as follows :—

TABLE III.

Estimated Mean Ordinates for Seven Groups.

Group.	6A.	Max.	2	3	4	5	6	7	8	9	10	11	12
I.	+1·0	0	13	38	61	87	100	94	70	40	30	17	10
II.	+0·5	0	12	32	54	78	97	98	83	56	39	26	11
III.	0·0	0	11	28	48	70	90	100	93	72	52	34	12
IV.	−0·5	0	10	26	44	63	82	98	100	86	65	42	14
V.	−1·0	0	9	24	40	57	76	93	100	94	77	53	20
VI.	−1·5	0	8	22	36	52	70	87	98	98	89	64	28
VII.	−2·0	0	7	20	32	48	65	80	94	99	98	75	38

6. We may now submit these mean curves to analysis and obtain the following coefficients.

TABLE IV.

Coefficients of Harmonic Terms for Curves of Table III.

Group.	6A	6B	6C	6D	6E	6F	$6A_4$	$6B_4$
I.	+1·07	−2·57	−0·46	+0·08	+0·02	−0·22	−0·01	+0·14
II.	+0·46	−2·62	−0·25	+0·10	0·00	−0·22	+0·03	+0·07
III.	−0·12	−2·79	−0·02	0·00	0·00	−0·20	+0·02	+0·04
IV.	−0·66	−2·75	+0·18	−0·16	0·00	−0·17	+0·07	+0·04
V.	−1·12	−2·58	+0·18	−0·36	+0·02	−0·19	+0·05	+0·06
VI.	−1·53	−2·33	+0·11	−0·54	+0·05	−0·23	+0·04	+0·08
VII.	−1·88	−2·05	−0·05	−0·69	+0·06	−0·38	+0·04	+0·13

The coefficients of the fourth harmonic, $A_4 \sin 4\theta + B_4 \cos 4\theta$, have also been calculated as a general check. By retaining the factor 6 (which occurs in the arithmetic), the range in magnitude is represented as 6, a larger range than usually occurs. Hence the fourth harmonic seldom makes a difference as large as 0·1 magnitude, and we may not unreasonably neglect it.

7. A curious point now arises. The angle θ in the expression

$$y = A \sin \theta + B \cos \theta + C \sin 2\theta + D \cos 2\theta + E \sin 3\theta + F \cos 3\theta$$

has been reckoned from a maximum, and hence we should have y a maximum when $\theta = 0$; the condition for which is

$$A + 2C + 3E = 0$$

But this condition is not fulfilled: in group VII., for instance, we have

$$6(A + 2C + 3E) = -1{\cdot}88 - 0{\cdot}10 + 0{\cdot}18 = -1{\cdot}80$$

It was at first supposed that there was some mistake in calculation; and to test this, the curve was recalculated (C) from its formula, excluding the fourth harmonic, for comparison with the assumed 12 ordinates (O) as below

	Max.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(C)	3	4	22	34	45	66	84	94	98	99	77	35
(O)	0	7	20	32	48	65	80	94	99	98	75	38
$(o-c)$	−3	+3	−2	−2	+3	−1	−4	0	+1	−1	−2	+3

There seems no reason to doubt the computations; but the observed and computed curves clearly do not fit closely at maximum. They intersect three times, and it would require a harmonic of a high order to obtain a good fit.

8. There is no reason why there should not be a harmonic of a high order; but neither is there any good reason why there should be. The necessity for it may be avoided by a simple supposition, but as yet we cannot say whether that supposition is true or false;

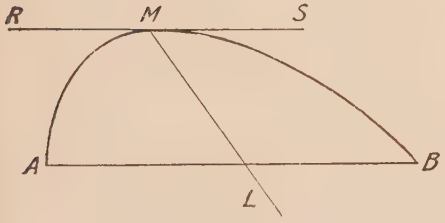

Fig. 1.

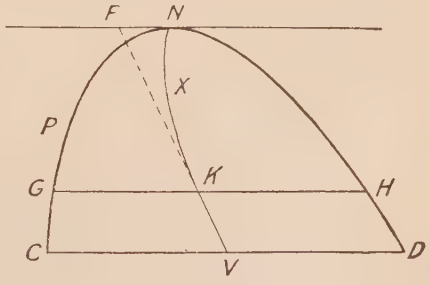

Fig. 2.

that can only be determined by a much more accurate knowledge of the shape of light-curves *close to maximum*. The supposition is that observers may have made a systematic error in assigning the epoch of maximum in cases where the rise and fall are not equally sharp. It is a common practice to assign this epoch by drawing the curve which bisects the chords joining equal magnitudes on the rise and fall; and it is now submitted that this practice cannot be defended, and may even be quite wrong. The reason will be clear from the above diagrams.

In fig. 1 is a portion of a parabola touching R M S at the point M. The locus of the middle points of parallel chords such as A B is, as we know, the straight line L M cutting the curve at the required point M. But it is quite easy to draw a curve, as in fig. 2, where the locus of these middle points is a curved line K X N, and where the portion V K produced would carry us towards a point F away from the true maximum. To realise this we have only to suppose the curve V K N drawn first along with one side C P N of the curve. We can then construct the other side by producing G K and similar lines to equal distances on the other side of V K N, when we shall get a smooth curve N H D.

9. We might have drawn V K X N much more curved, and still it might be made a locus of middle points of chords. The argument is of some importance, and may be put into an analytical form for comparison with our formulæ. We have adopted the formula

$$y = A \sin\theta + B\cos\theta + C\sin 2\theta + D\cos 2\theta + E\sin 3\theta + F\cos 3\theta \quad (1)$$

and near $\theta = 0$ this may be put into the form

$$y = \text{const} + (A + 2C + 3E)\theta - \tfrac{1}{2}(B + 4D + 9F)\theta^2 - \tfrac{1}{6}(A + 8C + 27E)\theta^3 + \text{etc.} \quad (2)$$

If the origin is a point of maximum, the constant is zero, and further

$$A + 2C + 3E = 0. \quad . \quad . \quad . \quad (3)$$

Thus the curve near the origin takes the form

$$y = b\theta^2 - a\theta^3 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $$a = \tfrac{1}{6}(A + 8C + 27E) = C + 4E \quad . \quad . \quad (5)$$

in virtue of the relation (3).

We may put this in the alternative form

$$b^2\theta^2 - ay\theta - by = 0 \quad . \quad . \quad . \quad (6)$$

which has the advantage of giving only two values of θ for any assigned value of y, excluding a third value of θ given by equation (4), at a finite distance from the origin and irrelevant. Now if these two values of θ be θ_1 and θ_2, the co-ordinates of the middle point of the chord joining them are (x, y), where

$$x = \tfrac{1}{2}(\theta_1 + \theta_2) = ay/b^2 \quad . \quad . \quad . \quad (7)$$

and the equation to the locus of middle points of chords near the origin is thus

$$y = \frac{b^2}{a}x = \frac{(B + 4D + 9F + \ldots)^2}{4(C + 4E + \ldots)}x \quad . \quad . \quad (8)$$

If there were only the first harmonic, all the coefficients C, D, E, F except A and B would be zero, and the coefficient of x in equation (8) would be infinite, so that the locus of middle points would cut the axis of x at right angles. If we take in the next harmonic, then since C and D are small compared with B, the coefficient of x is still very large, and the direction of the locus is *nearly* at right angles. But when we take in E and F, and more and more as we take in harmonics of higher order, they are multiplied by factors which increase rapidly and indefinitely; so that the sign and the magnitude of the coefficient of x in equation (8) are ultimately controlled entirely by harmonics of a high order, however small their coefficients may be. We cannot therefore assert *anything* about the ultimate direction of the locus unless we know from independent considerations what is the character of these harmonics of high order. The locus of middle points is not exactly a bad guide to the maximum (or minimum); it is simply no guide at all. The epochs should not be assigned in this way; * and if they have been so assigned in the past, they may be systematically wrong.

10. On the other hand, there is no way of ascertaining the error except by closer and more continued observations near maximum. The amount of possible error may be illustrated by determining the maxima in another way, viz. from the assumed formulæ. It is at least equally good (or bad) with the former, and a comparison of the two methods will show the kind of error to which either is liable.

11. It will suffice to give an approximate correction. In the neighbourhood of the origin we have

$$\begin{aligned} y &= (\mathrm{A} + 2\mathrm{C} + 3\mathrm{E})\theta - \tfrac{1}{2}(\mathrm{B} + 4\mathrm{D} + 9\mathrm{F})\theta^2 \\ &= \alpha\theta - \tfrac{1}{2}\beta\theta^2 \end{aligned}$$

Let y be a maximum when $\theta = \epsilon$. Then

$$\alpha - \beta\epsilon = 0$$

The values of α, β, and ϵ are given below in Table V. The deduced values of ϵ do not fit in very well with the hypothesis of a systematic error in estimating, for they are not symmetrical on opposite sides of group III.; in other words a greater error is made when the rise is sharp than when the fall is sharp, the reason for which is not apparent. But the phenomenon is of a progressive nature.

* Further consideration has led to the view that the characteristics of the curves must be determined in a totally different way, by calculating areas and centres of gravity and moments. But this discussion is reserved for a future paper.

TABLE V.

Calculation of Correction to Epoch of Maximum.

Group.	6α	6β	Ratio.	ϵ
I.	+0·21	−4·23	−0·050	−2°·9
II.	−0·04	−4·20	+0·009	+0·5
III.	−0·16	−4·59	+0·035	+2·0
IV.	−0·30	−4·92	+0·061	+3·5
V.	−0·70	−5·73	+0·122	+7·0
VI.	−1·16	−6·56	+0·177	+10·1
VII.	−1·80	−8·23	+0·219	+12·5
The Sun	−0·68	−6·83	+0·100	+5·7

12. When we alter the epoch of maximum by ϵ, the terms in 2θ are altered by 2ϵ and those in 3θ by 3ϵ. The new values of the coefficients are given in Table VI., which, on the hypothesis now made, should replace Table IV. The old value of 6A is inserted in the second column to show the change.

TABLE VI.

Corrected Coefficients of the Harmonic Terms.

Group.	6A	$6A_0$	$6B_0$	$6C_0$	$6D_0$	$6E_0$	$6F_0$
I.	(+1·07)	+0·96	−2·62	−0·45	+0·13	−0·01	−0·22
II.	(+0·46)	+0·49	−2·62	−0·25	+0·09	+0·01	−0·22
III.	(−0·12)	−0·02	−2·79	−0·02	0·00	+0·02	−0·20
IV.	(−0·66)	−0·49	−2·79	+0·20	−0·14	+0·03	−0·17
V.	(−1·12)	−0·80	−2·69	+0·26	−0·31	+0·08	−0·17
VI.	(−1·53)	−1·09	−2·57	+0·28	−0·47	+0·16	−0·17
The Sun	(−1·49)	−1·25	−2·44	+0·24	−0·71	+0·25	−0·13
VII.	(−1·88)	−1·39	−2·41	+0·28	−0·64	+0·28	−0·26

13. What is true of the groups is, of course, true of the determinations for individual stars; and if we return to Table II. and calculate the epochs of maximum from the coefficients given in Table I. (or for the Rousdon stars in *Memoirs R.A.S.*, vol. lv. p. xcix), we shall find different values for A. Those which follow the general law above noticed call for no further remark; but there are some which go the other way, so that their places in the series are considerably changed. The most notable cases are—

	ϵ	Old A	New A_0
S Urs. Maj.	+10°	+0·35	+0·84
R U Herculis	+17°	−0·54	+0·30
R V Herculis	− 4°	−1·04	−1·24
S Lyræ	− 5°	−1·10	−1·37
R Urs. Maj.	− 5°	−1·11	−1·38
S Coronæ	−10°	−1·15	−1·66

It was found, however, that nothing essentially new was added by making the individual corrections; and as it is by no means certain that they are justifiable, the corrected table need not be given.

14. The question now arises—Is there any physical reason for this gradation or sequence in the light-curves? The facts suggest a light-curve of essentially uniform type, modified by some secondary cause. The place of the Sun in the series suggests the consideration of solar phenomena as a possible type for all these variables. Is there, then, any way in which these phenomena would be slightly modified in the case of stars scattered at random?

15. One such source of modification is certainly the *difference in aspect*. We view the Sun from the direction of his equator, whereas the stars must present all aspects to us from pole to equator. What would happen to the Sun's light-curve (as represented by the spot-curve) if we viewed the Sun from the direction of one of its poles?

16. The general consequence is easily seen. Spots in high latitudes would always be near the centre of the apparent disc, while those near the equator would be considerably foreshortened. [The same would be the case with faculæ: it need not cause any misunderstanding if we speak of the more familiar spots throughout.] They would also suffer absorption near the limb; but for the present we will omit this fact, and think only of the foreshortening.

Beginning with the epoch of maximum, the subsequent spots, being in lower latitudes, would be more foreshortened and diminished in value, which would make the fall to minimum more rapid. Directly a few small spots appeared in high latitudes, their enhanced value would start the rise to maximum. But as the spots increased in number and size their latitude would diminish; they would leave the centre of the disc and become foreshortened. This would to some extent compensate the increase in number, and render the rise to maximum less steep.

17. The general effect above described is exactly that which would carry the Sun towards the other end of the series of light-curves. At present the fall is slow and the rise rapid; if we viewed the same phenomena from the direction of the Sun's pole, the fall would be more rapid and the rise slower. There remain the questions—What is the amount of the change? and how is the general character of the curve affected?

18. We will first consider the actual case of the Sun, though, for reasons which will presently appear, it is not a satisfactory example of the stars. The Greenwich Observations give the following values for the "mean distance from equator of all spots" in the years 1883–1904, arranged in two periods of eleven years.

1st Cycle	13°·1	11°·3	11°·8	10°·4	8°·4	7°·4	11°·6	22°·0	20°·3	18°·4	14°·5
2nd Cycle	14·2	13·5	14·3	8·0	10·5	9·5	7·7	10·4	17·6	19·9	16·6
Mean	13·6	12·4	13·1	9·2	9·4	8·4	9·7	16·2	19·0	19·2	15·6

For the same years the "corrected areas" are given as follows :—

1st Cycle	1155	1079	811	381	179	89	78	99	569	1214	1464
2nd Cycle	1282	974	543	514	375	111	75	29	62	340	488
Mean .	1219	1027	677	448	277	100	77	64	316	777	976

The factor for foreshortening when the spots are supposed to be viewed from the pole is sin λ, which is very nearly proportional to λ, and may be so taken for the present purpose. Multiplying the two rows of means we get to 3 significant figures,

166 127 89 41 26 8 7 10 60 149 152

To treat this as we have treated the light-curves we subtract 166 and divide by 159, reversing the signs so that maximum is represented by zero and minimum by +1·00, obtaining therefore

00 24 48 79 88 99 100 98 67 11 9

19. To compare with this we must take the "projected areas" as we see them from the direction of the equator. As given in the Greenwich volumes for the same years these are

1st Cycle	1595	1478	1122	527	243	125	103	133	745	1596	1983
2nd Cycle	1728	1330	745	695	532	159	101	41	86	434	653
Mean .	1662	1404	933	611	388	142	102	87	416	1015	1318

and treating these in the same way we get (after subtracting 1662 and dividing by 1575)

0 16 46 67 81 96 99 100 79 41 22

20. There are only eleven terms in each case instead of twelve. We might draw a curve and read off twelve points for analysis as before; but it seems better to take actual observations untouched, and the above numbers have been analysed on the assumption that the period is exactly eleven years. The results are as below:—

	6A	6B	6C	6D	6E	6F
From Pole .	+0·50	−3·06	+0·53	−0·26	−0·14	+0·28
From Equator .	−0·28	−2·82	+0·24	−0·47	−0·10	−0·04

21. There is thus a change of sign in A, and at first sight it would appear that we have obtained an effect of the kind required. But no attempt was made to assign the maximum accurately, and this is still to be done by the method given above, the values of $A+2C+3E$ being different from zero in both cases. The value for the second curve is small, and we find $\epsilon = +1^\circ\cdot2$. For the first, $A+2C+3E$ is so large that the approximate method for finding ϵ will not work, and we must find it by trial. The value $\epsilon = 27^\circ$ is near the truth; and on substituting it the value of 6A is changed from +0·50 to −0·95, which *is on the other side* of the

value for equator view. Indeed a rough sketch of the two curves without any analysis shows that they are not very different in type; and that although the spot minimum of the curve has been shifted, the maximum has been shifted even *more* in the same direction; so that the curve, instead of being changed from a "late" to an "early" type, is "later" than ever. The fact is that the peculiarity of the Sun already referred to in § 3 makes it for this purpose an unsuitable representative of variable stars.

22. Let us consider what are the conditions determining the shift of a maximum or minimum. In the neighbourhood of such a point, for which $\theta = \theta_0$ say, the ordinates are of the form

$$y_1 = \mathrm{K}(\theta - \theta_0)^2 + \text{higher powers}$$

If to these we add ordinates of the form

$$y_2 = \mathrm{G}(\theta - \theta_0) + \ldots$$

we shift the position of the maximum or minimum. The condition for it is

$$0 = \frac{dy_1}{d\theta} + \frac{dy_2}{d\theta} = 2\mathrm{K}(\theta - \theta_0) + \mathrm{G}$$

and the shift is thus G/2K provided G is small compared with K. The quantity G is the gradient of the second curve, and K represents the curvature of the first. The shift can be increased then by increasing the gradient of the second curve, *or* by diminishing the curvature of the first. Finally, if curvature and gradient be both reversed in direction, the shift remains unaltered in direction.

23. To apply this to the example just given, we must first take logarithms of the quantities used in order to substitute the addition of two ordinates for their multiplication. Let S be the spotted area and λ the spot latitude: we formed the product

$$x = \mathrm{S}.\lambda$$

Taking logarithms we get

$$y = \log x = \log \mathrm{S} + \log \lambda = y_1 + y_2 \quad \text{say}$$

and we are concerned with the alteration of the maxima and minima of y_1 by the gradient of y_2. Now the gradient of y_2 is small near a maximum of y_1, but sharp near a minimum, when the spots quickly cross into high latitudes. Thus in § 18 we see that the mean latitude changes near minimum from 9°·7 to 19°·0 in two years, the change in the reverse direction requiring seven or eight years. The gradient at minimum being thus much sharper, we might expect the shift of the minimum to be much greater than that of the maximum; and in the case of the stars we shall presently see that this probably happens. But in the particular case of the Sun the sharper gradient in y_2 is more than compensated by the greater curvature in y_1. This does not appear

in the spot numbers themselves; but when we take logarithms we get the following quantities, omitting a constant, viz.—

1·31 1·23 1·05 0·87 0·67 0·22 0·12 ·03 0·73 1·11 1·21

which represent a curve with a curvature at minimum much sharper than at maximum, which is not the case with the magnitude curves of long-period variables. This difference between the Sun and the stars is important for our purpose. It is the sun-spot numbers themselves, and not their logarithms, that resemble the magnitude curves of variables; and it was remarked above (§ 3) that they provide the proper analogy when the variation in brightness is small.

24. Another way of regarding the matter is to think of actual brightness curves for the stars. A rise of 5 magnitudes, for instance, means that the brightness at maximum is 100 times that at minimum. If we drew the curve with the ordinate at maximum 5 inches high, the ordinates near minimum would be very small The curve would be very flat near minimum, and shoot up into a sudden high peak near maximum with, of course, a sharp curvature. For the Sun this high peak is evanescent; or, if we magnify it to make it noticeable, the zero line of our light-curve recedes to a very great distance.

25. In considering the effects of foreshortening for the stars, we may deal with either the light-curves or the magnitude-curves. In the former case we multiply the ordinates by factors; in the latter we add other ordinates to them, representing the logarithms of the factors. But we must remember in either case that it is the faculæ only which are supposed foreshortened. Let P be the total brightness of the photosphere, F of the faculæ, and B of the two together, and let s be the factor for foreshortening. Then

$$B = sF + P$$

If we have two different aspects of the star for which the values of s are s_1 and s_2 respectively, then

$$\log B_1 - \log B_2 = \log (s_1F + P) - \log (s_2F + P)$$

Now, near maximum P is very small compared with sF, and we may neglect it. Thus

$$\log B_1 - \log B_2 = \log s_1 - \log s_2$$

26. Consider now the case of minimum. If the faculæ die out altogether, so that F = 0, then

$$\log B_1 - \log B_2 = 0$$

The minimum for both cases is then the same, as is indeed obvious.

If the faculæ do not entirely die out,

$$\log B_1 - \log B_2 = \log (s_1 + p) - \log (s_2 + p)$$

where p is written for P/F, and is positive. This difference is always less than $\log s_1 - \log s_2$, but approaches it as p diminishes, *i.e.* as the faculæ at minimum are brighter compared with the photosphere. It is not by any means impossible that if the immense increase in brilliancy of these stars is due to faculæ, then even at minimum there may be faculæ whose brightness largely exceeds that of the photosphere, so that we might write without sensible error

$$\log B_1 - \log B_2 = \log s_1 - \log s_2$$

throughout the whole light curve. For simplicity we shall consider this case first: the requisite modification for other cases can be considered later (§ 40). But we may again remark how the case of the Sun differs from that of the stars. For the Sun p is always a large quantity even at maximum; and a different formula is appropriate, viz.—

$$B_1 - B_2 = k(s_1 - s_2)F$$

27. We must now consider the factor s. When the Sun's or star's axis is tilted, the visibility of a spot is affected in two ways.

(*a.*) *Length of visible path.*—If the pole is towards us we see the whole path of a circumpolar spot; if the equator is towards us, only half of it. But if the spots are symmetrically distributed in both hemispheres, then, taking a pair of equal latitudes together, we always see just half the length of path of the spots in them. When a pole is towards us, we see the whole path in one hemisphere, none in the other; when the equator is towards us, we see just half of each. Hence the mean length of visible path is not altered by tilt of the axis.

(*b.*) *Foreshortening.*—Let λ be the latitude of a spot, ϵ the distance of the pole (P) of the star from the centre of the disc (Z, fig. 3). Then, denoting the angle S P H by Z, the angular distance ζ of the spot S from the centre of the disc Z is given by $\cos\zeta = \cos\epsilon \sin\lambda + \sin\epsilon \cos\lambda \cos Z$. The foreshortening factor is just this $\cos\zeta$, and the mean factor for the spot may be taken as

$$\int \cos\zeta \, dZ = [Z \cos\epsilon \sin\lambda + \sin\epsilon \cos\lambda \sin Z]$$

the expression in brackets being taken within limits, say from $-\alpha$ to $+\alpha$. Now if we take a pair of latitudes ($+\lambda$ and $-\lambda$) together, the limits for $-\lambda$ will be $-(\pi - \alpha)$ to $+(\pi - \alpha)$; and the total effectiveness will thus be

$$s = 2(2\alpha - \pi)\cos\epsilon \sin\lambda + 4 \sin\epsilon \cos\lambda \sin\alpha$$

α being given by the equation

$$\cos\alpha = -\cot\epsilon \tan\lambda$$

It will be more convenient to put $2\alpha - \pi = 2\beta$ and use the angle β. The equations become

$$\sin\beta = \cot\epsilon \tan\lambda$$

and $$s = 4\beta \cos \epsilon . \sin \lambda + 4 \cos \beta \sin \epsilon . \cos \lambda$$

in which β is to be put equal to $\frac{\pi}{2}$ for all values of λ greater than ϵ.

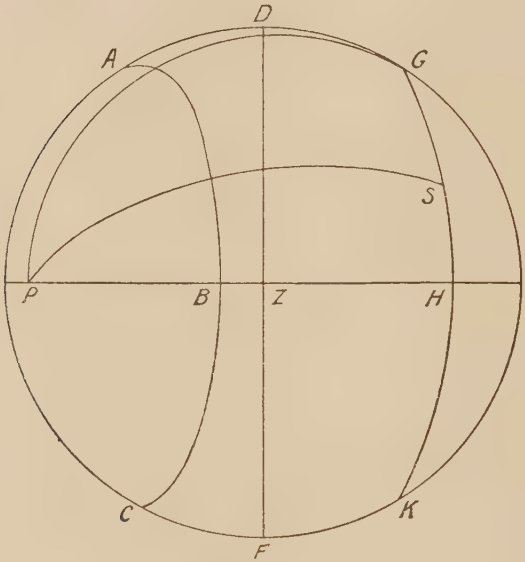

Fig. 3.

28. The extreme cases are simple:—

Equatorial View. Here $\epsilon = 90^\circ, \beta = 0^\circ$, and
$$s = 4 \cos \lambda$$
$$\log s = \log 4 + \log \cos \lambda$$

The log 4 may be disregarded as a constant addition to the light-curve. The term log cos λ can theoretically have any value from 0 to $-\infty$; but λ in practice is to be taken as a *mean* latitude, and is not likely to exceed (say) 70°; for which log cos $\lambda = 9{\cdot}53$. The range in log s is thus not more than 0·5. Note that in this case log s *decreases* as λ *increases.*

Polar View. Here $\epsilon = 0^\circ, \beta = 90^\circ$, and
$$s = 2\pi \sin \lambda$$
$$\log s = \log 2\pi + \log \sin \lambda$$

The range in log sin λ is again theoretically infinite; but the mean latitude of the spots is not likely to approach nearer the equator than say 5°, for which log sin $\lambda = 8{\cdot}94$. The range in log s is therefore little greater than unity, and it *increases* as λ increases.

29. *Intermediate Cases.* For other values of ϵ the formula is more complex, but the change in log s is intermediate between the above extreme cases. The general character of it can be readily seen as follows:—

Put $\beta \cot \gamma = \tan \epsilon \cos \beta$

then $s = 4 \cos \epsilon . \beta \operatorname{cosec} \gamma . \cos (\lambda - \gamma)$

When $\lambda = 0, \beta = 0$ and $\gamma = 0$

When λ is small

$$\beta = \lambda \cot \epsilon, \quad \gamma = \lambda \cot^2 \epsilon, \quad s = 4 \sin \epsilon \cos k\lambda, \quad \text{where} \quad k = \cot^2 \epsilon - 1$$

And when $\lambda = \epsilon, \beta = \frac{\pi}{2}$ and $\gamma = \frac{\pi}{2}$

and these latter values of β and γ are to be retained for all values of λ greater than ϵ.

Hence when λ is near 90°

$$s = 2\pi \cos \epsilon \sin \lambda$$

The case when $\epsilon = 45°$ will perhaps sufficiently illustrate the nature of the variation in log s. The values of β and γ are given below in degrees, but this need cause no misunderstanding.

TABLE VIII.

Reduction Factors when $\epsilon = 45°$.

λ °	β °	γ °	$\gamma - \lambda$ °	log cos $(\gamma - \lambda)$	log β cosec γ	log s = Sum.
0	0·0	0·0	0·0	0·00	0·00	0·00
10	10·1	10·1	0·1	0·00	0·00	0·00
20	21·3	21·9	1·9	0·00	0·00	0·00
30	35·2	37·0	7·0	0·00	0·01	0·01
35	44·4	47·3	14·3	9·99	0·02	0·01
40	57·0	61·3	21·3	9·97	0·06	0·03
45	90·0	90·0	45·0	9·85	0·20	0·05
50	90·0	90·0	40·0	9·88	0·20	0·08
60	90·0	90·0	30·0	9·94	0·20	0·14
70	90·0	90·0	20·0	9·97	0·20	0·17
80	90·0	90·0	10·0	9·99	0·20	0·19
90	90·0	90·0	0·0	0·00	0·20	0·20

It will be seen how rapidly β and γ change near the limit $\lambda = \epsilon$, introducing rapid changes in log cos $(\gamma - \lambda)$ and in log β cosec γ. But these are in opposite directions and practically compensate one another, so that the variation in log s (shown in the last column with a constant omitted) is small. This is what we might expect in the case midway between $\epsilon = 90°$, when log E decreases, and $\epsilon = 0°$, when it increases as λ increases.

30. We return to the extreme cases, and to the inquiry whether the available range is sufficient to convert one extreme type of magnitude-curve into the other. The conversion is made by the equation

$$\begin{aligned}\log B_1 - \log B_2 &= \log s_1 - \log s_2 + \text{const.}\\ &= \log \tan \lambda + \text{const.}\\ &= \log \tan \lambda - \log \tan \lambda_0\end{aligned}$$

where λ_0 is the latitude common to the two curves.

31. Referring now to Table III., let B_1 refer to group VII. and B_2 to group I.; and let us inquire what values of λ will convert one of these curves into the other.

First superpose the maxima.

The table is arranged on this plan, the assumed maxima falling under one another. Subtracting the twelve ordinates we get for I.–VII.

$$\cdot 00 + \cdot 06 + \cdot 18 + \cdot 29 + \cdot 39 + \cdot 35 + \cdot 14 - \cdot 24 - \cdot 59 - \cdot 68 - \cdot 58 - \cdot 28$$

These, however, are not differences of logarithms to base 10. We must multiply by about 4 to restore the range in magnitude, and by 0·4 the base of the magnitude scale, both factors together making 1·6. We must also reverse the signs, since increasing magnitudes mean decreasing brightness. Finally we must add the constant log tan λ_0; and since λ_0 is unknown, three suppositions are made, viz. $\lambda_0 = 10^\circ$, $\lambda_0 = 20^\circ$, $\lambda_0 = 30^\circ$, as follows:—

$\lambda_0 = 10^\circ$, log tan $\lambda_0 = 9{\cdot}25$

log tan λ = 9·25 9·15 8·96 8·78 8·62 8·69 9·03 9·63 10·20 10·34 10·18 9·70

$\lambda = 10^\circ 0\ \ 8^\circ 0\ \ 5^\circ 2\ \ 3^\circ 5\ \ 2^\circ 6\ \ 2^\circ 8\ \ 6^\circ 1\ \ 23^\circ 1\ \ 58^\circ 0\ \ 65^\circ 4\ \ 56^\circ 6\ \ 26^\circ 5$

$\lambda_0 = 20^\circ$, log tan $\lambda_0 = 9{\cdot}56$

log tan λ = 9·56 9·46 9·27 9·09 8·93 9·00 9·34 9·94 10·51 10·65 10·49 10·01

$\lambda = 20^\circ 0\ \ 16^\circ 0\ \ 10^\circ 6\ \ 7^\circ 0\ \ 4^\circ 9\ \ 5^\circ 8\ \ 12^\circ 6\ \ 41^\circ 0\ \ 73^\circ 0\ \ 77^\circ 6\ \ 72^\circ 0\ \ 46^\circ 0$

$\lambda_0 = 30^\circ$, log tan $\lambda_0 = 9{\cdot}76$

log tan λ = 9·76 9·66 9·47 9·29 9·13 9·20 9·54 0·14 0·71 0·85 0·69 0·21

$\lambda = 30^\circ 0\ \ 24^\circ 6\ \ 16^\circ 4\ \ 11^\circ 0\ \ 7^\circ 6\ \ 9^\circ 0\ \ 19^\circ 1\ \ 54^\circ 1\ \ 79^\circ 0\ \ 82^\circ 0\ \ 78^\circ 5\ \ 58^\circ 4$

32. The smallest assumed value of λ_0 brings the spots at minimum too close to the equator, while the largest takes them too close to the pole. In every case the range required is too large to be probable, though not impossible. There are, however, some considerations which reduce the range. We have superposed the observed maxima of Table III., but the time of maximum will certainly be altered by a change of aspect, as well as the time of minimum. Further, there is the uncertainty about the real time of maximum. Both these can be taken account of by tracing the

curves on paper and superposing them experimentally, and then reading off new points on one of them to correspond with the twelve points already tabulated for the other.

33. *Superposing the curves empirically*, then, in the neighbourhood of maximum, we get new readings for group VII. as follows, group I. remaining as before :—

	Max.											
VII.	·04	·16	·28	·42	·60	·74	·90	·98	·99	·83	·48	·18
I.	·00	·13	·38	·61	·87	1·00	·94	·70	·40	·30	·17	·10
I.–VII.	–·04	–·03	+·10	+·19	+·27	+·26	+·04	–·28	–·59	–·53	–·31	–·08

The range is now from +·27 to –·59 instead of from +·39 to –·68 before. Multiplying by 1·6 to convert into logarithms to base 10 we find the values of λ when $\lambda_0 = 20°$ to be now

22°·6 22°·2 14°·0 10°·3 7°·6 7°·9 17°·6 45°·8 73°·0 68°·8 49°·0 26°·0

The requisite range in latitude is thus reduced by 4°·6 near the pole (from 77°·6 to 73°·0) and 2°·7 near the equator (from 4°·9 to 7°·6), and the improbability therefore sensibly diminished.

34. A much greater diminution can be made if we take into account the effect of absorption of the light near the star's limb, in addition to the simple foreshortening. In the absence of any information as to its amount, it is impossible to go beyond a hypothetical illustration. But, as an illustration merely, if the reduction factor for foreshortening and absorption combined were $\cos^2 \zeta$ instead of the previous $\cos \zeta$ for foreshortening alone, then we must replace log tan λ in the above calculations by 2 log tan λ; in other words, we may halve the differences 1·6 (I.–VII.) before finding log tan λ from them.

For the case $\lambda_0 = 20°$ the values of λ would now be (for the empirical superposition),

21°·3 21°·0 16°·8 15°·1 12°·6 12°·6 18°·8 31°·7 47°·0 44°·5 33°·0 23°·0

But it is interesting to take another value of λ_0, viz. that which would be applicable in the case of the Sun. Putting $\lambda_0 = 11°·3$, the values of λ would be

12°·0 11°·8 9°·4 8°·0 7°·0 7°·0 10°·6 18°·6 30°·5 28°·0 19°·5 13°·0

and we may compare with these the values given in the Greenwich volumes for 1882 and the following eleven years for the "mean distance from equator of all spots," viz.—

17°·9 13°·1 11°·3 11°·8 10°·4 8°·4 7°·4 11°·6 22°·0 20°·3 18°·4 14°·5

Thus a range not much greater than that of the spots on the Sun would, on this hypothesis as to the reduction factor, suffice for converting one of the extreme cases in Table II. or Table III. into the other extreme.

35. There are a few stars outside these limits, as we shall see in § 36, § 37, so that a greater range is required. But it is tolerably clear that, in default of information as to the magnitude of absorption, we can neither affirm nor deny the sufficiency of this cause to explain the whole range of variation in type of light-curve.

36. As regards this range, one point remains to be considered. How far do the thirty-one stars of Table I. cover the range in type? Are there other stars much "earlier" or "later" than these? The best information available is that given in Chandler's Third Catalogue and the revised edition, where the quantity $M - m$, being the number of days by which maximum follows minimum, is tabulated alongside the period. If $2\ (M - m)$ exceeds the period, the star is an "early" star like T Cassiopeiæ: if it is less than the period, the star is "late" like the Sun. We may take $\alpha = \{2\ (M - m) - P\}/P$ as a characteristic of the star. Now by calculation we find the minimum of group I. in Table VI. falls at $180° - 18°$, so that $\alpha = +·10$: and the minimum of group VII. falls at $180° + 45°$, so that $\alpha = -0·25$. But these refer to intervals between *calculated* maxima and minima, which differ systematically from those assigned by the observer, as already pointed out. If we took the observer's maximum we should increase α numerically in both cases; and if we assume that the same will be true also at minimum, we increase α still further. It would not be unreasonable to double the numerical range for α, and consider that in Chandler's lists we may expect a range of

$$+·20 \text{ to } -·50$$

corresponding to the range of $+·10$ to $-·25$ adopted in comparing with the hypothesis.

37. In the first instance Chandler's Third Catalogue was taken, and the value of

$$\alpha = \{2\ (M - m) - P\}/P$$

formed for all variables of period greater than 100 days. The results were as follows:—

Table IX.

α	No of Stars.	α	No. of Stars.
$> +·20$	3	$-·21$ to $-·30$	12
$+·20$ to $+·10$	5	$-·31$ to $-·40$	4
$+·09$ to $·00$	12	$-·41$ to $-·50$	0
$-·01$ to $-·10$	38	$< -·50$	1
$-·11$ to $-·20$	28	Total	103

Thus four cases out of one hundred and three fell outside the limits above assigned. But on afterwards referring to Chandler's "Revision of Elements" (*Astronomical Journal*, 1904, Jan. 8), it was found that two cases out of the four had been altered so as to fall within the limits, as the particulars given below will show.

TABLE X.

Name.	No.	3rd Cat.			Revision.		
		M − *m*	P	α	M − *m*	P	α
S Leonis	3994	125	190d	+·32	93	189	−·02
W Scorpii	5795	146	222	+·32	130	221	+·18
V Cephei	8591	220	360	+·22	220	360	+·22
S Tauri	1582	70	376	−·63	70	380	−·63

Further, as regards S Tauri, it is to be remarked that Chandler gives twenty-one maxima but only two minima as the basis of elements, and possibly further observation may alter the value of M − *m*: and V Cephei only varies a magnitude at most, so that the epochs of maximum and minimum are not very easy to determine. The elimination of two exceptional cases out of four in the "Revision" of elements is not without significance, and we may perhaps expect further modifications in the future. On the other hand, one exceptional star has been introduced in the Revision (X Ophiuchi, see below), a star with small range in magnitude, and subject to the same criticisms as V Cephei.

TABLE XI.

Twenty Stars with Large Values of α.

α	Name.	No.	R.A. h m	Dec. °	Gal. Lat.
+·25	X Ophiuchi	6682	18 34	+ 9	12
+·22	V Cephei	8591	23 52	+83	23
+·19	R S Libræ	5511	15 18	−23	25
+·18	W Scorpii	5795	16 6	−20	18
+·17	V Ophiuchi	5887	16 21	−12	20
+·16	S Carinæ	3637	10 6	−61	12
+·13	S Arietis	715	1 59	+12	42
+·12	T Cassiop.	107	0 18	+55	5
+·11	T Capricorni	7659	21 17	−16	46
+·10	S Cephei	7779	21 36	+78	23
+·08	T Cephei	7609	21 8	+68	18
+·08	V Tauri	1717	4 46	+17	13
+·07	X Capricorni	7577	21 3	−22	44
+·07	W Cygni	7754	21 32	+45	13
+·07	U Boötis	5338	14 50	+18	56
+·05	V Cygni	7428	20 38	+48	10
+·05	R Camelop.	5190	14 25	+84	35
+·03	R Aurigæ	1855	5 9	+53	15
+·03	W Cassiop.	294	0 49	+58	5
+·02	R Sagittarii	6905	19 11	−19	16

TABLE XII.

Twenty Stars with Small Values of α.

α	Name.	No.	R.A.	Dec.	Gal. Lat.
			h m	°	°
−·63	S Tauri	1582	4 24	+10	20
−·42	R Androm.	112	0 18	+38	20
−·35	R Gemin.	2528	7 1	+23	20
−·34	S Coronæ	5504	15 17	+32	55
−·34	R Comæ	4315	12 0	+19	75
−·31	R Cancri	2946	8 11	+12	25
−·27	R Urs. Maj.	3825	10 38	+69	48
−·27	V Lyræ	6871	19 5	+30	5
−·27	R Cygni	7045	19 34	+50	15
−·27	U Cassiop.	243	0 41	+48	10
−·26	R Centauri	5095	14 9	−59	5
−·25	o Ceti	806	2 14	−3	51
−·23	R Can. Min.	2539	7 3	+10	12
−·22	Y Virginis	4492	12 29	−4	54
−·22	S Hydræ	3170	8 48	+3	28
−·22	R Aquilæ	6849	19 2	+8	0
−·20	U Orionis	2100	5 50	+20	0
−·20	V Bootis	5194	14 26	+39	65
−·19	S Piscium	434	1 12	+8	50
−·19	T Arietis	976	2 43	+17	32

In Tables XI. and XII. are collected the stars with extreme values of α at both ends of the series. Further observations of the light-curves of some of these would be specially welcome.

38. If the present hypothesis, which classifies the stars according to the orientation of their axes, be correct, it is natural to inquire whether any peculiarity of distribution in space is suggested. In the last column are given the Galactic Latitudes of the stars. It will be noticed that while the stars with large negative values of α occur in all Galactic Latitudes, those with positive values do not occur near the Galactic Poles. In terms of the hypothesis above sketched, this would be the case if the axes of rotation of these stars were roughly parallel to the plane of the Milky Way. We should then see stars equatorially in all directions, but we should not have a polar view of those lying near the Galactic Poles.

It may be remarked that the axis of the Sun is inclined about 30° to the plane of the Milky Way. It is not difficult to suggest physical reasons for such a general configuration, in view of the recent discoveries of two stellar systems in relative motion; but the material is so small that it would be laying too much stress on the figures to enter on such speculations. At the same time they are sufficiently suggestive to warrant further investigation.

39. We have now followed out the consequences of the assumption which first suggested itself, leaving, however, one important point for further consideration (§ 26), to which we must

return. But before doing so it will be convenient to recapitulate the main features of the hypothesis above dealt with, and to point out a limitation not hitherto noticed. The chief features are as follows:—

(*a*) The travelling of the star's activity in latitude will modify the apparent variation according to the aspect presented to the observer.

(*b*) The travelling is represented by a moderate gradient G near maximum, and a sharper gradient g near minimum, when the activity leaves the equator and breaks out in high latitudes.

(*c*) Both maximum and minimum are displaced in consequence. The amount of displacement depends on the gradient and the curvature near the maximum (or minimum). Near maximum the log (activity) is represented by a curve,

$$y_1 = \mathrm{K}(\theta - \theta_0)^2$$

and the log (foreshortening) by a curve

$$y_2 = \mathrm{G}(\theta - \theta_0)$$

and the shift of maximum is G/2K. Using small letters for minimum, the shift is $g/2k$.

(*d*) Now K and k are opposite in sign but nearly equal numerically. G and g are opposite in sign but g is greater. Hence maximum and minimum will be displaced in the same direction, but the minimum more than the maximum. Relatively to the maximum the minimum is displaced by

$$g/2k - \mathrm{G}/2\mathrm{K} = (g - \mathrm{G})/2\mathrm{K} \quad \text{approximately.}$$

And the effective displacement thus depends roughly on the excess of g over G.

(*e*) This excess depends on the fact that the spots travel from a high latitude λ_1 to a low latitude λ_2 in a time t_1, and back over the same distance in a much shorter time t_2. Let $t_1 = nt_2$, then roughly we shall have $g = n\mathrm{G}$, and effective displacement
$= (n-1)\mathrm{G}/2\mathrm{K}$
$= (n-1) \times$ displacement of maximum.
To get a large range in effective displacement, therefore, $(n-1)$ should be large, and therefore t_2 small.

(*f*) But now comes the limitation not hitherto noticed. The period t_2 is a superior limit for the displacement of minimum; for outside t_2 the gradient would be no longer g but G, which is opposite in sign. We thus require t_2 to be *large*, which conflicts with paragraph (*e*).

The value of $\alpha = \{2(\mathrm{M} - m) - \mathrm{P}\}/\mathrm{P}$ according to Chandler's Catalogues ranges at least from $+\cdot22$ to $-\cdot40$: that is, the minimum may fall anywhere between $\cdot39\mathrm{P}$ and $\cdot70\mathrm{P}$ after maximum.

$$\text{Thus} \qquad t_2 \ll \cdot31\mathrm{P} \qquad t_1 \gg \cdot69\mathrm{P}$$
$$\therefore\ n \gg 2\cdot2 \qquad n - 1 \gg 1\cdot2$$

If we accept the view that the observer's assignments of maxima

and minima are systematically erroneous, in the manner indicated in §§ 7–13, the range for α may be reduced. If we reduce it to one-half (which is probably more than we are entitled to do), we get

$$t_2 \lessdot \cdot 16P \qquad t_1 \gtrdot \cdot 84P$$
$$\therefore\ n \gtrdot 5\cdot 3 \qquad n-1 \gtrdot 4\cdot 3$$

It is really straining the possibilities to give n so large a value as 4. For one thing, we are allowing nothing for the change from gradient G to gradient g; and for another, we can scarcely admit so great a systematic error in the estimates of maximum and minimum. Probably n cannot exceed 3, and thus the displacement of minimum will be three times that at maximum, and the effective displacement about twice that at maximum. It follows that too favourable a case was taken both in § 31 and in § 33. In the former the maxima were superposed, which makes the curves I. and VII. fit more easily than when both maximum and minimum are displaced. In the latter, the fit was adjusted to be even easier.

It is true that these and other difficulties in stretching the hypothesis to suit the facts can all be removed by assuming a large enough effect for absorption of light near the limb in addition to the foreshortening, but there is a natural reluctance to draw upon an unknown factor of this kind.

40. We now return to § 26, where it was remarked that if the faculæ die out altogether at minimum, the epoch of minimum will not be altered at all. The alteration in type of curve will then be entirely due to the displacement of maximum. This is a complete inversion of our hypothesis: the "early" stars will now be those regarded from the equator, and the "late" stars those regarded from one of their poles. Since the Sun is regarded from the equator and is a "late" star, there is at first sight an inconsistency here; but it has been already pointed out how the case of the Sun differs from that of the stars, owing to the rudimentary nature of the variation. So far as actual calculation goes, this new hypothesis fits the facts for the Sun better than the old; for it was found in § 21 that on regarding the Sun from one of his poles, his curve became "later" than ever, owing to the great displacement of maximum.

41. We can quickly run through the work again and make the necessary modifications. Returning to Table III., we now inquire what additions to the curve I. will transform it into curve VII. when the *minima* are superposed. The following superposition is sufficiently exact to bring out the main features:—

I.	0	13	38	61	87	100	94	76	40	30	17	10
VII.	32	48	65	80	94	99	98	75	38	0	7	20
I.–VII.	−32	−33	−27	−19	−7	+1	−4	−5	+2	+30	+10	−10

The difference becomes nearly zero near minimum, owing to the faculæ dying out and leaving only the photosphere. The differences

from zero need not concern us at present; we are chiefly concerned with the variation near maximum, and we see that the differences are smaller than in § 31, ranging only from +30 to −33, instead of from +39 to −68. Thus log tan λ changes by 1·6 × ·63 = 1·0 instead of by 1·6 × 1·07 = 1·7; and λ need only change from 73° to 18° instead of (say) from 78° to 5°. But we are met by the difficulty that this change from 73° to 18° must occur in about *one quarter of the period near maximum*, which is too improbable. If we now make the same assumption about absorption as before, viz. that it converts the foreshortening factor cos θ into cos^2 θ, or log tan λ into 2 log tan λ, then the range in λ becomes 45° to 18°; or 30° to 10° would do. Even this seems a large change in latitude to take place in one-quarter of the period. It is not impossible, but it seems unlikely. The balance of probabilities seems to lie with the former hypothesis.

42. We can also invert these hypotheses in another way, by assuming that the maximum is a time of few spots and the minimum of many. But we should then find it difficult to get large changes in the epochs. At minimum the photosphere must be practically covered with spots, so that the mean latitude could not vary much; and at maximum there would be so few that their situation could make little difference.

Thus at present the first hypothesis seems preferable.

43. One other point may be briefly mentioned. When this idea of the variation in light-curve with aspect of the star first presented itself, it was thought that it might lead to an explanation of Chandler's periodic terms. He gives, for instance, the epochs of maximum of R Andromedæ as

$$2400141 + 410{\cdot}3\,\mathrm{E} + 30 \sin (12^\circ\,\mathrm{E} + 90^\circ)$$

the mean period being 410·3 days, but successive maxima being gradually displaced until they fall thirty days later than at first, and then back until they are thirty days earlier, and so on periodically. If the axis of rotation of the star were not fixed, but had a free period of conical oscillation, its aspect to us would change, and such variations in the epoch of maximum would follow. But we should apparently have to assume very large oscillations in some well-authenticated cases; and there are some, such as S Serpentis, for which Chandler gives

$$368{\cdot}5\,\mathrm{E} + 116 \sin (4^\circ\,\mathrm{E} + 6^\circ)$$

which the foregoing work shows that we could not explain at all. The origin of these periodic terms is more likely to lie in the existence of several periodicities of spot variation with varying amplitudes, such as Professor Schuster has recently discovered in the case of the Sun.

On the Jupiter Evection Term. By P. H. Cowell, M.A., F.R.S.

In *Monthly Notices*, vol. lxiv. p. 417, and again in vol. lxv. p. 145, I have shown that the observed value of the coefficient of the Jupiter Evection term in the Moon's longitude was $0''\cdot89 + 0''\cdot23 = +1''\cdot12$.

In an interesting paper in the *Astronomical Journal*, No. 592, Professor Newcomb states that he has calculated the theoretical coefficient of this term as $1''\cdot15$, whereas Hill and Radau had previously agreed in giving $0''\cdot89$.

The removal of the only serious discordance between observed and theoretical coefficients for short terms of a period clearly differing from the periods of possible errors of observation strengthens the case stated in the *Monthly Notices*, vol. lxvi. p. 306, for Le Verrier's mass of Venus against the recent reduction by 2 per cent.

Observations of Minor Planets from Photographs taken with the 30-inch Reflector of the Thompson Equatorial at the Royal Observatory, Greenwich, during the year 1905.

(*Communicated by the Astronomer Royal.*)

The following positions of minor planets were obtained from photographs taken with the 30-inch Reflector during the year 1905.

The plates were measured with the astrographic micrometer. Four reference stars were, as a rule, measured with the planet, their positions being derived when possible from the Catalogues of the Astronomische Gesellschaft.

The positions given are not corrected for Parallax.

log Parallax Correction = log Parallax Factor − log Δ.

Date and G.M.T. 1905.	Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d h m s	h m s	° ′ ″		′
	(407) Arachne.			
Jan. 10 10 30 43	5 51 53·74	+27 14 9·3	−7·556	+0·554
	(78) Diana.			
Jan. 17 11 42 14	6 23 31·99	+35 8 29·1	+9·105	+0·410
	(71) Niobe.			
Jan. 17 11 25 46	7 44 5·90	+35 56 50·1	−8·785	+0·372

Date and G.M.T. 1905.	Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d h m s	h m s	° ′ ″		
(270) Anahita.				
Jan. 17 11 58 24	8 43 15·76	+15 5 32·2	−8·978	+0·719
(374) Burgundia.				
Mar. 9 10 50 3	9 58 16·04	−0 34 10·9	−∞	+0·839
(148) Gallia.				
Feb. 25 12 29 16	10 20 12·02	+16 14 45·7	+8·697	+0·704
Mar. 2 10 30 5	10 16 17·94	+17 11 47·3	−9·037	+0·698
3 10 51 44	10 15 30·60	+17 23 7·7	−8·824	+0·692
(92) Undina.				
Feb. 25 12 4 23	11 45 1·98	+15 6 36·7	−9·113	+0·722
Mar. 2 10 45 0	11 41 46·68	+15 37 31·9	−9·330	+0·731
3 11 16 48	11 41 4·70	+15 43 48·2	−9·206	+0·720
22 9 46 25	11 27 30·85	+17 22 50·4	−9·218	+0·704
30 9 19 27	11 22 9·74	+17 50 55·2	−9·176	+0·697
(487) Venetia.				
Mar. 30 10 11 41	11 52 23·75	+15 51 57·3	−9·058	+0·713
Apr. 6 10 22 35	11 47 13·59	+16 19 36·8	−8·635	+0·703
(350) Ornamenta.				
Apr. 6 10 50 45	11 48 42·65	+37 35 59·9	−∞	+0·316
8 10 11 58	11 47 19·90	37 32 3·4	−8·765	+0·326
12 11 23 18	11 44 42·39	37 20 46·8	+9·076	+0·348
(42) Isis.				
Mar. 22 10 13 9	12 10 56·26	+13 13 28·9	−9·271	+0·747
27 10 8 24	12 6 13·42	+13 40 45·4	−9·198	+0·739
30 10 33 27	12 3 24·23	+13 55 16·5	−8·982	+0·730
Apr. 5 10 56 56	11 57 57·41	+14 19 12·7	−8·062	+0·723
(28) Bellona.				
Mar. 22 10 22 59	12 20 39·52	+8 0 10·9	−9·264	+0·786
27 10 23 28	12 16 52·64	+8 40 53·4	−9·174	+0·779
30 10 50 22	12 14 37·46	+9 3 40·2	−8·937	+0·773
Apr. 6 9 55 29	12 9 41·22	+9 49 57·2	−9·083	+0·768

Date and G.M.T. 1905.				Apparent R.A.			Apparent Dec.			Log. Parallax Factor. R.A.	Dec.
d		h	m	s	h	m	s	°	′	″		
						(163) Erigone.						
May	5	10	5	42	12	52	32·41	+0	12	13·1	+7·982	+0·834
	8	10	56	51	12	51	6·68	+0	20	43·5	+9·042	+0·835
						(334) Chicago.						
May	5	10	53	54	13	58	6·56	−5	37	36·8	−8·285	+0·867
						(26) Proserpina.						
May	5	11	21	37	14	31	33·43	−14	29	8·0	−8·448	+0·903
	8	11	44	35	14	28	49·23	−14	21	29·2	+8·518	+0·903
	9	11	7	42	14	27	57·02	−14	19	2·8	−8·296	+0·903
	10	10	44	38	14	27	4·98	−14	16	37·0	−8·693	+0·902
	22	10	58	57	14	17	34·27	−13	51	6·5	+8·827	+0·900
						(386) Siegena.						
June	23	10	36	6	15	42	6·50	+7	31	19·4	+8·980	+0·785
	26	12	2	16	15	40	42·39	+7	22	5·6	+9·373	+0·796
	27	11	39	4	15	40	17·57	+7	18	46·5	+9·326	+0·794
						(470) Kilia.						
June	28	10	34	23	16	49	24·46	−9	41	11·3	+8·210	+0·885
						(8) Flora.						
June	14	11	51	44	17	51	37·82	−19	9	15·0	−8·711	+0·917
	14	12	3	17	17	51	37·21	−19	9	15·8	−8·506	+0·920
	19	11	37	25	17	45	59·09	−19	19	14·0	−8·507	+0·918
	19	11	50	27	17	45	58·59	−19	19	17·4	−7·962	+0·918
	22	11	18	9	17	42	34·77	−19	25	30·8	−8·580	+0·918
						(19) Fortuna.						
June	23	11	31	16	18	26	4·32	−21	1	20·8	−8·920	+0·920
	26	12	27	11	18	22	55·60	−21	2	1·4	+8·575	+0·922
July	7	10	45	15	18	11	39·00	−21	4	32·8	−8·648	+0·922
						(478) Tergeste.						
July	8	10	41	51	18	26	58·15	−9	45	53·6	−8·810	+0·885

Date and G.M.T. 1905. d h m s	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log. Parallax Factor. R.A.	Dec.
(46) Hestia.				
June 23 11 4 54	18 50 12·05	−18 58 12·3	−9·214	+0·908
July 7 11 7 31	18 36 36·14	−19 12 8·4	−8·674	+0·917
8 11 6 43	18 35 37·64	−19 13 19·4	−8·608	+0·918
25 10 19 33	18 20 59·57	−19 36 26·0	+8·229	+0·919
(313) Chaldæa.				
July 7 11 34 45	19 5 7·92	−5 16 15·9	−8·667	+0·865
19 11 59 11	18 53 55·98	−6 0 58·0	+8·934	+0·868
25 11 11 53	18 48 54·29	−6 29 32·3	+8·749	+0·871
(433) Eros.				
July 19 13 18 52	21 25 5·15	−13 49 49·5	−8·471	+0·901
Aug. 23 9 31 50	20 24 9·19	−12 33 0·5	−8·874	+0·895
(176) Idunna.				
Sept. 7 11 26 34	21 30 4·67	+12 16 26·9	+9·001	+0·746
(248) Lameia.				
Sept. 7 12 0 35	21 53 7·36	−5 44 6·8	+9·062	+0·866
(84) Clio.				
Sept. 8 12 16 24	22 37 56·92	−2 27 38·5	+8·882	+0·850

The anonymous planet discovered on photographs taken on 1903 Aug. 6, 31 and Sept. 1 has received the designation 1903 LX[a], and a provisional circular orbit has been calculated for it by Professor Kreutz (*Astron. Nachr.*, No. 4154).

Royal Observatory, Greenwich:
1907 *March* 7.

A New Nebula. By the Rev. T. E. Espin, M.A.

On January 18, while looking for new double stars, I came across a bright nebula, which, as far as I am aware, is an unrecorded object. It was estimated to be about 5″ in diameter and elongated north. On January 27 the nebula was seen for a few moments between clouds. On February 1 it was well seen, a conspicuous object, and equal to a 10 magnitude star. The elongation was very marked, and sometimes it looked like two nebulæ. The measures gave—

Major Axis	6″·90
Minor Axis	6·35

The major axis was found to be roughly at position 10°·5. A small star was noted S *p*. On February 11 the position was determined from B.D + 33°,746. It was found to precede this star by 7ˢ·80, and to be 2′ 25″ south of it. On February 18 what appeared to be a star or nucleus was seen north and measured. It was also seen on February 22 and February 23, and measures made of it with great difficulty. The later observations seem to suggest a planetary nebula, with a small star at the northern edge. The following are my measures of the two stars near the nebula :—

Neb. and star A :—

1907·1314	P 14°·9	D 4″·10		Mag. 13·0
1424	13·3	4·75		
1451	13·3	3·90		
Mean 1907·140	13·8	4·25	3 nts.	

Neb. and star B :—

1907·0849	244·6	16·75		Mag. 12·0
1314	244·3	17·05		
1424	245·3	15·90		
1451	238·4	17·05		
Mean 1907·126	243·1	16·69	4 nts.	

The place of B.D + 33°·746 for 1855·0 is—

$\alpha = 3^h\ 47^m\ 14^s{\cdot}89$ $\delta = 33^\circ\ 29'\ 14''{\cdot}7$ (*Bonn Obs.*, vol. vi.)

Professor Burnham observed the nebula with the 40-in. of the Yerkes Observatory on February 20 in moonlight and a sky not clear, and measured the star B as follows :—

P 238°·1 D 17″·86

and also found for B.D + 33°·746 and neb.

P 213°·9 D 177″·87 single distance.

Observations of Occultations. By Rev. Leonard A. Williams, B.A.

(*Communicated by A. C. D. Crommelin.*)

First Series.

At Stoke Wake Rectory, Blandford, Dorset.

Longitude, 2° 20′ 5″ W.

Latitude, 50 51 20 N.

Height above sea, 440 feet.

Star and Magnitude.	Disappearance G.M.T.				
		d	h	m	s
A Leonis 4·6	1905 May	12	8	44	27
γ Tauri 3·9	Sept.	19	10	35	40
Solar Eclipse—					
First con.	Aug.	29	23	45	19
Last con.		30	2	13	45

Second Series.

At Farningham Vicarage, Dartford, Kent.

Longtitude, 0° 13′ 25″ E.

Latitude, 51 22 45 N.

Height above sea, 120 feet.

Star and Magnitude.	Disappearance G.M.T.				
		d	h	m	s
μ Ceti 4·4	1905 Dec.	8	5	34	26
χ Leonis 4·7	1906 April	6	7	3	27
o Sagittarii 3·9	Nov.	19	5	29	30
ξ^2 Ceti 4·3	1907 Jan.	21	9	5	9
ν Geminorum 4·1		26	6	39	55

Method of determining time.—At Stoke Wake: Equal altitudes with sextant, and artificial horizon (mercury). At Farningham: Meridian observations with transit instrument.

Chronometer.—Deck watch in box, No. 08354, by Frodsham. Rate per diem for Jan. 1907, $-\cdot 9^s$.

Results.—The times are not *guaranteed* to the exact second; but they are assumed, with good reason, to be correct to 1·5 seconds.

1907 *January* 30.

Appendix to the above Paper. By A. C. D. Crommelin.

I have reduced Mr Williams' observations in the same manner as the Greenwich observations of occultations. The R.A., N.P.D., and parallax of Moon have been interpolated with second differences from the Nautical Almanac, and the semidiameter found by the equation log semidiameter = log parallax + 9·43542, this being the constant to reduce to Struve's semidiameter 15′ 32″·65, which is now used at Greenwich in the reduction of occultations. The R.A. and N.P.D. of the stars are taken with the Nautical Almanac values of their mean places and star corrections.

Then assuming—

True Time of Phenomenon = Observed Time + t^s

,, R.A. of Moon = Naut. Alm. value + $x'' + t \times$ mot$^{n.}$ in 1^s

,, N.P.D. . . . = ,, + $y + t \times$ mot$^{n.}$ in 1^s

,, R.A. of Star = ,, + e

,, N.P.D. . . . = ,, + f

,, Parallax of Moon = ,, $\times \left(1 + \frac{m}{1000}\right)$

,, Semidiameter ,, = Naut. Alm. Par$^{x.}$ $\times \cdot 272535 \times \left(1 + \frac{n}{1000}\right)$

I have found the following equations of condition; in the case of occultations observed at Greenwich, the Greenwich value of Semid. – Distance is given for comparison :—

Date.	Semid.—calculated Distance. Williams.	Greenwich.	$=e\times$	$+f\times$	$+x\times$	$+y\times$	$+t\times$	$+m\times$	$+n\times$
1905 May 12	−7″·40		=+·77	+·62	−·77	−·61	−·43	−·60	−·89
Sep. 19	−0·91		=+·11	−·99	−·11	+·99	−·19	+2·17	−·96
Dec. 8	−4·18		=+·78	−·61	−·78	+·61	−·42	+·04	−·88
1906 Ap. 6	−6·36	−7·81	=+·51	+·86	−·51	−·86	−·37	−2·97	−·97
Nov. 19	−2·33	−5·12	=+·92	+·18	−·92	−·18	−·46	+·69	−·99
1907 Jan. 21	−1·56		=+·99	+·05	−·99	−·05	−·36	+1·24	−·90
26	−3·20	−4·68	=+·87	+·37	−·87	−·37	−·35	−2·18	−·89

[The following were received too late for insertion in the Annual Report of the Council.]

Report of the Melbourne Observatory. (*Director Mr P. Baracchi.*)

The principal astronomical work done at this observatory during the year 1906 was limited, as in previous years and for the same reasons, to meridian observations and stellar photography, including the measurement of plates of the Sydney and Melbourne Zones, in regard to which a separate report is appended.

Meridian Observations.—These were made with the 8-inch Transit Circle, and were as follows :

	Observations in R.A.	Observations in N.P.D.
Clock Stars . . .	505	...
Azimuth Stars . . .	269	117
List Stars	1276	1289
Total .	2050	1406

The list stars were selected from the Melbourne plates of the Astrographic Catalogue, to serve as fundamental points of reference for the reduction of these plates.

The total number of this class of stars now completely observed not less than three times is 5545.

The reductions, including the preparation of the annual catalogue for 1905, are well advanced.

A general catalogue for the epoch 1900, including all stars observed since 1894, is in course of preparation.

No authority has yet been obtained for printing the general catalogue for the epoch 1890, the MS. of which was prepared some years ago.

Stellar Photography.—The Melbourne portion of the photographic catalogue and chart of the heavens has been further advanced as follows :—

	Passed as satisfactory.	Rejected.
Chart Plates with triple exposure of 30^m each .	45	3
Catalogue Plates (duplicate series)	19	...
Test Plates on South Polar region	14	...
Test Plates on Oxford Type Charts . . .	5	...
Plates for trials, adjustment of centre, etc. . .	15	...

Prolonged ill-health of the observers is partly accountable for the small progress shown by the above return.

The astrophotographic work now stands thus :—

The first series of 1149 plates, covering the entire region, $-64°$ to the South Pole, twice, and the series of chart plates with single exposure of one hour, the centres being at even degrees of declination from $-66°$ to $-90°$, were completed some time ago.

A duplicate catalogue series is now being made, and of this, 335 plates have been taken and passed as satisfactory.

In the chart series with triple exposures of 30^m each, the centres being at odd degrees of declination, 519 plates have been taken and passed as satisfactory; 65 plates require to be taken again to conclude this part of the work.

It is intended to extend the series of triple exposure chart plates to the regions with centres at even degrees of declination.

The following routine duties and other miscellaneous work were carried out for local requirements, as in former years :—

The time service; the weather service, comprising the control of some 980 country stations; the rating of chronometers for the shipping, and the testing of nautical meteorological and surveying instruments; the operations of the Bureau of Standard Weights and Measures; the continuous registration of the variations of sea-level, atmospheric elements, seismic disturbances, and the elements of terrestrial magnetism, including absolute magnetic measurements and the measurement and reduction of hourly ordinates on the magnetic curves of past years, for the purpose of clearing up, completing, and preparing for publication the results of a long series of magnetic records, extending back to the year 1868.

Joint Report of the Directors of the Observatories of Sydney and Melbourne on the Measurement of the Plates of the Astrophotographic Catalogue.

The measurement of catalogue plates obtained at these observatories was continued by the Bureau established for this purpose at the Melbourne Observatory in 1898, and since maintained at the joint expense of the States of New South Wales and Victoria.

The work of the year 1906 was carried out by the usual temporary staff of six young ladies, assisted by a permanent officer of the Melbourne staff, using the same measuring instruments, and following in every respect the same methods as described in reports of former years.

The numbers of rectilinear co-ordinates measured during the year in the direct and reverse positions of the plates are as follows :—

149 Sydney Plates, containing 75,162 stars.
181 Melbourne Plates, containing 45,069 stars.

The total aggregate numbers of plates, etc. measured to 31st December 1906 are as follows :—

562 Sydney Plates, containing 325,978 stars.
836 Melbourne Plates, containing 268,714 stars.

ERRATUM IN ANNUAL REPORT.

On p. 298 insert the name of Prof. E. E. Barnard in list of Donors to the Library.